Plenary talks

The Solvability of Differential Equations

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Since Hans Lewy presented his famous counterexample sixty years ago, it has been known that almost all non-symmetric linear partial differential equations are not solvable. For differential equations with simple characteristics, solvability is equivalent to the Nirenberg-Treves condition (Ψ). This condition involves the non-symmetric part of the highest order terms.

In this talk, we shall consider differential operators that have double characteristics. Then the solvability may depend on the lower order terms, and one can define a condition corresponding to (Ψ) on these terms. We shall show that this condition is necessary for solvability in several cases.

The role of Banach Gelfand Triples in the context of Conceptual Harmonic Analysis

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The idea of "Conceptual Harmonic Analysis" grew out of the attempt to make objects arising in Fourier Analysis or Gabor Analysis (such as norms of functions, their Fourier transforms, dual Gabor atoms, etc.) computable. Using suitable function spaces such as the Segal algebra $S_0(R^d)$ it should be possible to find concrete algorithms which allow to compute approximations on existing hardware in finite time, up to (at least potentially) arbitrary requested precision. Going beyond the ideas of Abstract Harmonic Analysis, which only allows to identify the analogies between objects on different LCA (locally compact Abelian) groups, the idea of Conceptual Harmonic Analysis is to emphasize the connections between these settings, for example, in order to use methods from discrete, periodic Gabor analysis (computationally realizable using e.g. MATLAB) in order the study the continuous case.

The Banach Gelfand Triple $(S_0, L^2, S_0')(R^d)$, which can also be seen as a *"rigged Hilbert space"*, justifies a number of such procedures providing a tool to deal with periodic, or continuous or discrete signals in a unified way. As it turns out the so-called $w*$-convergence is crucial, and fine partitions of unity as well as regularizing operators play a crucial role in this context. Due to the *kernel theorem* for this setting also various spaces of operators can be studied.

[1] Elena Cordero, Hans G. Feichtinger, and Franz Luef. Banach Gelfand triples for Gabor analysis. In *Pseudo-differential Operators*, Vol. 1949 of *Lecture Notes in Mathematics*, Springer (2008), 1–33.

[2] Hans G. Feichtinger, *Elements of Postmodern Harmonic Analysis.* In *Operator-related Function Theory and Time-Frequency Analysis. The Abel Symposium 2012, Oslo, August, 2012*, Springer (2015), 77–105.

Fractional-order operators: Boundary problems, heat equations

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The fractional Laplacian $P = (-\Delta)^a$, $0 < a < 1$, — and other fractional-order operators — has been studied intensively in recent years because of its interesting applications in mathematical physics and differential geometry, and in probability

and finance (in the latter because it is the infinitesimal generator of a stable Lévy process). It is a linear operator, entering also in nonlinear problems. Some of the questions that are asked are similar to those asked in connection with the Laplacian, but now with the additional difficulty that $(-\Delta)^a$ is a nonlocal operator.
The trend among nonlinear PDE people has mostly been to use methods from potential theory and singular integral operators, whereas the fact that it is a pseudodifferential operator has not been taken much into account, until recently. We shall talk about some of the results that can be obtained via pseudodifferential methods, for boundary value problems, e.g., the homogeneous Dirichlet problem defined from a variational setup, but also an inhomogeneous Dirichlet problem with nontrivial boundary data, and Neumann-type problems. Most recently, regularity results have been shown for problems with a time-parameter, the associated heat equation $Pu(x,t) + \partial_t u(x,t) = f(x,t)$.

[1] Gerd Grubb, *Fractional Laplacians on domains, a development of Hörmander's theory of μ-transmission pseudodifferential operators*, Adv. Math. **268** (2015), 478–528.

[2] Gerd Grubb, *Integration by parts and Pohozaev identities for space-dependent fractional-order operators*, J. Diff. Eq. **261** (2016), 1835-1879.

[3] Gerd Grubb, *L_p-regularity of solutions to fractional heat equations*, arXiv:1706.06058.

On traffic modeling and the Braess paradox

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We will discuss models for vehicular traffic flow on networks. The models include both the Lighthill–Whitham–Richards model and Follow-the-Leader models. The emphasis will be on the Braess paradox in which adding a road to a traffic network can make travel times worse for all drivers.

The talk is based on collaborations with N. H. Risebro (Oslo) and R. Colombo (Brescia)

[1] H. Holden, N. H. Risebro. Continuum limit of Follow-the-Leader Models. *arXiv 1702.01718.*

[2] H. Holden, N. H. Risebro. A mathematical model of traffic flow on a network of unidirectional roads. *SIAM J. Math. Anal.* **26** (1995), 999–1017.

[3] R. Colombo. H. Holden. On the Braess paradox with nonlinear dynamics and control theory. J. Optim. Theory Appl. **168** (2016), 216–230.

Dyadic representation of singular integrals and applications

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Singular integral operators play a prominent role in various areas of pure and applied analysis. Due to the singular nature of the defining integrals, it is often preferable to find alternative representations with better convergence properties. A particular class of such representations of high recent interest consists of averages over families of so-called *dyadic shifts*. A breakthrough of this method was the solution of A_2 *conjecture* concerning sharp quantitative norm bounds for singular integrals on weighted $L^2(w)$ spaces [3], but some of its roots (in a different but not unrelated form) go back to the work of Beylkin et al. on numerical algorithms for the computation of singular integrals [1]. Nowadays there are simpler proofs of the A_2 conjecture, where the linear identity given by the dyadic representation is replaced by a non-linear upper bound in terms of so-called dyadic domination, but the dyadic representation continues to have other applications, where linearity is required, e.g. in the estimation of *commutators* of singular integrals and pointwise multipliers [2]. Aside from the existing results, I might speculate a little about the possibility of adapting these dyadic representations – very efficient for theoretical considerations – also for numerical computations, in the spirit of [1].

[1] G. Beylkin, R. Coifman, V. Rokhlin, *Fast wavelet transforms and numerical algorithms I*, Comm. Pure Appl. Math. **44** (1991), 141–183.

[2] I. Holmes, M. Lacey, B. Wick, *Commutators in the two-weight setting*, Math. Ann. **367** (2017), 51–80.

[3] T. Hytönen, *The sharp weighted bound for general Calderón–Zygmund operators*, Ann. of Math. **175** (2012), 1473–1506.

[4] T. Hytönen, *Representation of singular integrals by dyadic operators, and the A_2 theorem*, Expo. Math. **35** (2017) 166–205.

Spectral inequalities and their applications

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I shall briefly describe the theory of coherent state transformations and its applications to the spectral theory. Then I apply this theory to the study of the Weyl type asymptotics and spectral estimates of functional-difference operators associated to mirror curves of special del Pezzo Calabi-Yau threefolds.

Solvability of Planar Complex Vector Fields With Elliptic Degeneracies

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Let $L = A(x, y)\partial_x + B(x, y)\partial_y$ be a vector field in $\mathbb{R}^2$ with $\mathbb{C}$-valued coefficients A and B. When L is elliptic in a domain $\Omega \subset \mathbb{R}^2$, a change of variables reduces the study of L to that of $\overline{\partial}$. The goal of this presentation is to consider the case when L has elliptic degeneracies in an open set Ω. That is, when the set $\{p \in \Omega;\ L \wedge \overline{L} = 0\}$ is non empty. Sufficient conditions for the solvability of L will be given. The main tool used for the solvability is a generalized Pompeiu operator for L. Riemann-Hilbert type boundary value problems for L will also be considered.

Almost-positivity of pseudodifferential operators

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It is well-known that a priori estimates such as the Gårding, the Sharp-Gårding, the Melin, the Hörmander and the Fefferman-Phong inequalities play a central role in a variety of problems, ranging from spectral theory, well-posedness of the Cauchy problem of parabolic or hyperbolic type (especially in presence of degeneracy, i.e. multiple characteristics), to local and global solvability of PDEs, hypoellipticity and subellipticity, just to mention a few. In my talk I will in the first place recall the mentioned inequalities and explain the influence of the geometry of the characteristic set and of lower-order terms in the estimates; I will then discuss generalizations and open problems in some directions (higher characteristics, relaxation of the geometric assumptions, non-Euclidean settings) and finally describe what is known, at present, about systems of pseudodifferential operators.

Morrey spaces from various points of view

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Let $1 \leq q \leq p < \infty$. Define the *Morrey norm* $\| \star \|_{\mathcal{M}^p_q}$ by

$$\|f\|_{\mathcal{M}^p_q} \equiv \sup \left\{ |Q|^{\frac{1}{p}-\frac{1}{q}} \|f\|_{L^q(Q)} \, : \, Q \text{ is a cube in } \mathbb{R}^n \right\}$$

for a measurable function f. The *Morrey space* $\mathcal{M}^p_q(\mathbb{R}^n)$ is the set of all measurable functions f for which $\|f\|_{\mathcal{M}^p_q}$ is finite.

This norm goes back to the paper by Morrey in 1938 [1]. Later based on the observation by Morrey Peetre started to consider the Morrey norm [2].

We are oriented to the following topics:

(1) The origin of Morrey spaces–the observation made by C. Morrey.
(2) Examples of the functions in Morrey spaces.
(3) Morrey spaces from the viewpoint of functional analysis.
(4) Interpolation of Morrey spaces.
(5) Fourier transform and Morrey spaces.
(6) Weighted Morrey spaces.
(7) Various related spaces.

The talk is based on collaborations with many people, whose name will be listed in the talk.

[1] C.B. Morrey, *On the solutions of quasi linear elliptic partial differential equations,* Trans. Amer. Math. Soc. **43** (1938), 126–166.

[2] J. Peetre, *On the theory of* $\mathcal{L}_{p,\lambda}$, J. Funct. Anal. **4** (1969), 71–87.

Applications of dynamical systems theory in biology

Organizers: T. Lindström, A. Asta, L. Tamburino

Directed preferential attachment models

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The original preferential attachment model for an undirected network was 10 years ago generalized to a directed network model, where nodes together with out- or in-directed edges appear sequentially, and where the probability to attach to a given node is proportional to the in- (out-) degree of that node. The bivariate degree distribution is defined as the in- and out-degree of a randomly selected node. In the talk we analyse the model using a new approach where it is shown that the distribution can be characterized by two independent birth processes observed at a common exponentially distributed time-point. The method is easily extended to more realistic models allowing for doubly-directed edges as well as attachment probabilities depending both on in- and out-degree.

Stochastic Epidemic Model Driven by white noise or Lévy jumps

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There are many cases when deterministic models are not adequate. For example, dynamics fluctuations are not smoothed out by statistical averaging, and the time evolutions of such systems are therefore stochastic. The randomness in the system usually cannot be ignored, thus, one is forced to adopt a stochastic description. the stochastic models take into account in addition of the mean trend, the variance structure around it.

In this work we are interested in the study of the behavior of the global positive solution for an epidemic model characterized by temporary immunity. We analyse the qualitative behavior of the disease around both the disease-free and endemic equilibria. We show that the solution does random fluctuations with an intensity related to the values of the volatility or jump increments.

The talk is based on collaborations with B. Berrhazi, A. Laaribi and R. Taki.

Modelling and analysis of maturation processes of stem cell populations

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I will first present a pde model for the regulated maturation process of a maturity structured stem cell population [2]. Integration along the characteristics leads to a differential equation with state-dependent delay. I will show some recent results on global well-posedness and the associability of a linear variational equation that allows stability analysis [1,3]. There will also be presented work in process on local and global stability of equilibria.

The talk is based on collaborations with Istvan Balazs, Tibor Krisztin, Yukihiko Nakata, Gergely Röst and Marcus Waurick.

[1] O. Diekmann, Ph. Getto and Y. Nakata, *On the characteristic equation* $\lambda = \alpha_1 + (\alpha_2 + \alpha_3\lambda)e^{-\lambda}$ *and its use in the context of a cell population model* Journal of Mathematical Biology **72(4)** (2016) 877–908.

[2] Ph. Getto and A. Marciniak-Czochra, *Mathematical modelling as a tool to understand cell self-renewal and differentiation* book chapter in M dM. Vivanco (ed.) "Mammary stem cells - Methods in Molecular Biology" Springer Protocols, Humana Press, 2015.

[3] Ph. Getto and M. Waurick, *A differential equation with state-dependent delay from cell population biology*, Journal of Differential Equations **260(7)** (2016) 6176–6200.

Long-term behaviour of deterministic population models with stochastic interventions

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For understanding the long-term dynamics of some biological population dynamics one needs to consider models in the class of so-called piece-wise deterministic Markov processes. In such models, during a period of time the system evolves according to a deterministic description (e.g. given by an ordinary or partial differential equation), followed by a stochastic jump in state space. The probability distribution for timing or size of this jump, or both, can be dependent on the state of the system. Such models occur for example when considering antibiotic treatment of bacterial infection, fishing, or insect populations with non-overlapping generations when the stochastic effects on death in winter are taken into account.
In the presentation we will introduce these examples, motivate the modeling approach and discuss analysis techniques for studying the long-term behaviour of the corresponding models. In the latter we take an analytical (Markov) operator point of view. That is, we study the dynamical system in the space of probability measures defined by iteration of these operators.

Cycles and multiple attractors explained in terms of saturation effects for mixotrophy

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In this paper we check the dynamical consequences of the invasion of mixotrophs by the use of a model that is a limiting case of the explicit resource dynamics modeling of the chemostat. We first consider the unsaturated case and end up with a rather complete description of its dynamical behavior in terms of its mixotrophic parameters. Indeed, the model has always at least one locally stable equilibrium.
The saturated case is more difficult to describe and saturation causes the possibility for a unique limit cycle in the predator-prey plane and multiple equilibria in the competition plane. We show that the invasion of mixotrophs has a stabilizing impact on autotroph-herbivore dynamics.

The talk is based on collaboration with Yuanji Cheng and Subhendu Chakraborty.

[1] C. Jost, C. A. Lawrence, F. Campolongo, W. van de Bund, S. Hill, and D. L. DeAngelis. *The effects of mixotrophy on the stability and dynamics of a simple planctonic food web model.* Theoretical Population Biology, **66**, (2004), 37-51.

How to find efficient harvesting strategies in stage-structured populations

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We show how to find harvesting strategies that account for both yield and conservation when allowing for different harvesting rates on juveniles and adults. Our methods are based on construction of several measures of conservation as well as Pareto-efficient trade-offs between these measures and yield. By applying our technics on two well established population models we show that (1) pretty good yield can give large conservation benefits, (2) equal harvesting rates of juveniles and adults is often a good strategy, and that (3) the impact of harvesting on population biomass and size structure acts as warning signals of an impending collapse.

The talk is based on collaborations with Nicolas Loeuille, Xinzhu Meng, Mats Bodin and Åke Brännström.

A Subgradient Method for Unconstrained Nonconvex Nonsmooth Optimization

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In this talk, we present an algorithm for solving non-convex non-smooth optimization problems by using Clarke Subdifferential. We propose an iterative method that solves an unconstrained nonconvex nonsmooth optimization problem. The proposed method is a descent method that uses subgradients at each iteration and contains very simple procedures for finding descent directions and for solving line search subproblems. This method can be applied to solve linearly constrained problems.

Modelling molecular motor function for drug discovery

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Cell motility, e.g. muscle contraction,is driven by cyclic interactions between the proteins actin (A) and myosin II (M), powered by turnover of the phosphonucleotide ATP. The latter is converted to the hydrolysis products ADP (D) and inorganic phosphate (Pi) on the myosin active site and then sequentially released, coupled with structural changes. Disturbances of the actin-myosin interaction occur in e.g. heart diseases, cancer and ageing. This has inspired drug development but the efforts are hampered by limited understanding of the actin-myosin interaction. First, results of studies using isolated actin and myosin in solution, often central in drug discovery, do not self-evidently report how a drug affects force-generation by ordered actin-myosin ensembles. Second, the central process in actin-myosin force-generation from attachment of myosin to actin, via Pi-release to the force-generating structural change, is poorly understood. Here, a recent model [1] is expanded to help overcoming the challenges. The model integrates coarse-grain structural information with elastic energies in the actomyosin-links and transition rates from biochemical studies. Predictions for actin-myosin ensembles on different scales are derived by solving differential equations in state probabilities and by Monte-Carlo simulations. The model approximates physiological properties of muscle e.g. maximum shortening velocity, maximum isometric force and force response to stretch. Also the relationship between actin-concentration and ATP turnover rate is accounted for. Furthermore, hypotheses for the molecular mechanisms of action of two existing drugs yield predictions in agreement with experiments. The results elucidate central aspects of force- and motion-generation by actin and myosin. Furthermore, the prediction of ensemble behavior on basis of parameters from studies of isolated proteins will be of great value in drug-discovery.

[1] A Månsson, *Actomyosin based contraction: one mechanokinetic model from single molecules to muscle?*, J. Muscle Research Cell Motility, **37** (2016) 181-194.

Hypersensitive Optimal Control of Invasive Species

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Effectively dealing with invasive species is a pervasive problem in environmental management. The damages, and associated costs, that stem from invasive species are well known, as is the benefit from their removal. We investigate problems where optimal control theory has been implemented, and we show that these problems can easily become hypersensitive, making their numerical solutions unstable. We show that transforming these problems from state-adjoint systems to state-control systems can provide useful insights into the system dynamics and simplify the numerics. We apply these techniques to two case studies: one of feral cats in Australia, where we use logistic growth; and the other of wild-boars in Italy, where we include an Allee effect. A further development is to optimize the control strategy by taking into account the spatio-temporal features of the invasive species control problems over large and irregular environments. The approach is used in a management scenario where the invasive species to be controlled with an optimal allocation of resources is the deciduous tree *Ailanthus Altissima*, infesting the Alta Murgia National Park in the south of Italy.
This work has been carried out within the H2020 project ECOPOTENTIAL (http://www.ecopotential-project.eu), coordinated by CNR-IGG. The project has received funding from the European Union's Horizon 2020 research and innovation programme (grant agreement No 641762).

The talk is based on collaborations with C.M. Baker, F. Casella, D. Lacitignola, F. Diele, C. Marangi.

[1] Baker C.M., *Target the source: Optimal spatiotemporal resource allocation for invasive species control*, Conservation Letters, February 2016.
[2] Baker C.M., D. Lacitignola, F. Diele, C. Marangi, A. Martiradonna, *Hypersensitive Optimal control of invasive species through a dynamical systems approach* , submitted.

Numerical bifurcation analysis of delay models for physiologically structured populations

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Physiologically structured populations are often described by mathematical models where a renewal equation for the birth rate is coupled with a delay differential equation for the environmental variable. These equations generate dynamical systems on an infinite-dimensional function space.
In order to study numerically the bifurcation properties when varying some parameters, we apply the pseudospectral discretization technique to the infinite-dimensional system and obtain a finite-dimensional system of ordinary differential equations. The discretized system is easy to write from the original equation and ensures high-accuracy approximations with low system dimension, thanks to the spectral convergence properties of the approximation scheme. Finally, the

bifurcation properties of the approximating system can be numerically studied with standard software for ODEs, with no need of developing tailor-made software for delay equations.
This procedure has proved to be effective on systems with finite delay [1, 2] and has recently been extended to equations with infinite delay. Numerical examples support the flexibility of the approach.

The talk is based on collaborations with Mats Gyllenberg (University of Helsinki, Finland) and Rossana Vermiglio (University of Udine, Italy).

[1] D. Breda, O. Diekmann, D. Liessi, F. Scarabel, *Numerical bifurcation analysis of a class of nonlinear renewal equations*, Electronic Journal of Qualitative Theory of Differential Equations **65** (2016), 1–24.

[2] D. Breda, O. Diekmann, M. Gyllenberg, F. Scarabel, R. Vermiglio, *Pseudospectral discretization of nonlinear delay equations: new prospects for numerical bifurcation analysis*, SIAM Journal on applied dynamical systems **15** (2016), 1–23.

Mathematical model of hormonal treatment of cancer under hormonal resistance

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There are certain forms of cancer associated with the action of hormones. The hormone therapy applies for this case. However, the effect of hormonal treatment is only temporary because of the gradual habituation of cancer cells to the action of hormones. The tumor cells consists of two types. There are sensitive and resistant cells to the action of hormones. The resistant cancer cells substitute the sensitive cells by the hormonal treatment. Therefore, a further development of the tumor is observed.
We propose the mathematical model of the hormonal resistance phenomenon. This model is based on the Volterra competition model and one-phase Stephan problem. This is the system of partial differential equations

$$\frac{\partial u_s}{\partial t} = D_s \frac{\partial^2 u_s}{\partial x^2} + \Big[\frac{a_s}{1+f(t)u_s^\theta} - b_s(u_s+u_r)\Big] u_s + c_{rs}u_r,\ 0 < x < \xi(t),\ t > 0;$$

$$\frac{\partial u_r}{\partial t} = D_r \frac{\partial^2 u_r}{\partial x^2} + \big[a_r - b_r(u_s+u_r)\big] u_r + c_{sr}u_s,\ 0 < x < \xi(t),\ t > 0;$$

where u_s and u_r are the concentration of the sensitive and resistant cancer cells, D_s and D_r are its diffusion coefficients, a_s and a_r characterize its increase, b_s and b_r take into account the limitations of the environment, c_{sr} and c_{sr} describe the transformations of cells, f and θ determine the concentration of treatment and its effectiveness, and ξ describes the size of the tumor. We suppose that the sensitive calls are more viable, if the treatment is not used. It is known also the initial values the sensitive and resistant cancer cells, besides, the sensitive calls are prevailed. We suppose the source of the tumor on the fixed left boundary, and the zero value of the cancer cells on the variable right boundary. We have also the analogue of Stephan condition on the variable boundary The numerical analysis of the proposed mathematical model is realized.

The talk is based on collaborations with D. Nurseitov and A. Azimov.

Behaviour of many predators - one prey systems

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Ecological modeling is becoming increasingly more important for all kinds of modern applications. The mathematical language of dynamical systems has been applied since ancient times. In this talk we introduce and discuss some main methods for studying dynamical systems, mainly for the analysis of nonlinear systems of many predators feeding on one prey. We show how important results can be obtained by simple methods that are based on elementary mathematics. We include both simple and different types of chaotic behaviour.Most models of predators and preys indicate cycles where populations are becomming unrealisticly small. We point out that Deterministic Models are heavily criticised, e.g. amongst Swedish specialists in stochastics. On the other hand, subarctic biologists confirm that predators are not behaving stochasticly, but rather switching feeding between species. This leads to dynamical systems with switches, well known in engineering applications. To conclude, we will also mention some challenging open problems in this subject.

Approximation Theory and Special Functions - 4th Series

Organizers: O. Duman, E. Erkus-Duman

Approximation by nonlinear integral operators with the help of summation methods

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We study the approximation properties of nonlinear integral operators of convolution-type introduced in [1] with the help of a summability process which is a general convergence method introduced by Bell (see [2]). In the approximation we use a semi-norm defined on the space of all periodic functions with bounded variation on an interval. We compute rates of approximation and also discuss some special cases.

The talk is based on collaborations with Professor Oktay Duman.

[1] L. Angeloni and G. Vinti, *Convergence in variation and rate of approximation for nonlinear integral operators of convolution type,* Result. Math. **49** (2006), 1-23.

[2] H. T. Bell, *Order summability and almost convergence*, Proc. Amer. Math. Soc. **38** (1973), 548-552.

Riesz Potential in the Local Morrey-Lorentz Spaces

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In this study, the necessary and sufficient conditions are found for the boundedness of the Riesz potential I_α in the local Morrey-Lorentz spaces $M^{loc}_{p,q;\lambda}(\mathbb{R}^n)$. This result is applied to the boundedness of particular operators such as the fractional maximal operator, fractional Marcinkiewicz operator and fractional powers of some analytic semigroups on the local Morrey-Lorentz spaces $M^{loc}_{p,q;\lambda}(\mathbb{R}^n)$.

The talk is based on collaborations with V.S.Guliyev, A. Kucukaslan and A. Serbetci.

[1] C. Aykol, V.S. Guliyev and A. Serbetci, *Boundedness of the maximal operator in the local Morrey-Lorentz spaces*, Jour. Inequal. Appl., 2013, 2013:346.

[2] C. Bennett and R. Sharpley, *Interpolation of Operators,* Academic Press, Boston, 1988.

[3] A. Cianchi, R. Kerman, B. Opic, L. Pick, *A sharp rearrangement inequality for the fractional maximal operator.* Studia Math., **138** (2000), 3, 277-284.

[4] N. Samko, *Weighted Hardy and potential operators in Morrey spaces.* Journal of Function Spaces and Applications 2012, DOI: 10.1155/2012/678171.

[5] Samko N. *Weighted Hardy and singular operators in Morrey spaces.* J. Math. Anal. Appl., **350** (2009) 56–72.

On statistical approximation properties of a genuine type (p,q)-Phillips operators

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In this speech, we consider a genuine type modification of (p, q)-analoque of the Phillips operators given in [1,2]. Firstly, we give the construction of these operators and general formula for calculating the test functions which will be used in evoluating the moments of them. Then, we explore the local approximation theorems of a genuine (p, q)-Phillips operators. Afterward, we establish a statistical weighted approximation theorem and find the statistical rate of convergence of them. Furthermore, we study Voronovskaja type theorem and get direct results.

The talk is based on collaborations with Professor İsmet Yüksel.

[1] C. P. May, *On Phillips operators*, J. Approx. Theory **20** (4) (1977), 315–332.

[2] İ. Yüksel, *Approximation by q− Phillips operators*, Hacet. J. Math. Stat. **40** (2) (2011), 191–201.

Some convergence methods on pseudo linear operators

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We study approximation to a non-negative continuous function by means of pseudo linear operators including max-min and max-product operators, where the concept of pseudo linearity weakens the usual linearity (see Bede et.al. [1]). In the approximation, we consider not only the ordinary convergence but also some weaker convergence methods, such as statistical convergence, arithmetic mean convergence, almost convergence (see [2,3]). Following these results we examine the error estimation via the modulus of continuity. At the end of the presentation, we give some applications to indicate why we need these operators.

The talk is based on collaborations with Professor Oktay Duman.

[1] B. Bede, H. Nobuharab, M. Dankova and A. Di Nola, *Approximation by pseudo-linear operators*, Fuzzy Sets and Systems **159** (2008), 804-820.

[2] O. Duman, *Statistical convergence of max-product approximating operators*, Turkish J. Math. **34** (2010), 501-514.

[3] T. Y. Gokcer and O. Duman, *Summation process by max-product operators*, In Computational Analysis, Springer Proc. Math. Stat. **155** (2016), 59-67.

Approximation by generalized Bleimann-Butzer-Hahn operators of max-product kind

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The nonlinear Bleimann-Butzer-Hahn operators of max-product kind proposed in [2] and studied intensively in [1]. We give a generalization of these operators which are constructed using the sequences of positive real numbers and we obtain a quantitative type theorem for these operators.

The talk is based on collaborations with Professor Nurhayat İspir.

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Weighted approximation by q-Ibragimov-Gadjiev operators

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The weighted approximation of continuous functions by q-Ibragimov-Gadjiev operators introduced in [3] is studied. In addition, rate of approximation of these operators is obtained with the help of the weighted modulus of continuity introduced by Gadjiev (see[1,2]).

The talk is based on collaborations with Professor İbrahim Büyükyazıcı.

[1] A. D. Gadjiev, *The convergence problem for a sequence of positive linear operators on unbounded sets and theorems analogous to that of P.P. Korovkin,* Soviet Math. Dokl. **15** (1974), 1433-1436.

[2] A. D. Gadjiev, *On P. P. Korovkin's theorems,* Math. Z. **20** (5) (1976), 781-786.

[3] S. Herdem and İ. Büyükyazıcı, *Ibragimov-Gadjiev Operators based on q-integers,* Submitted for publication, 2017.

Extended multivariable hypergeometric functions

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In this paper, we introduce extension of multivariable hypergeometric functions by using similar method in [1,2]. Then, we obtain some generating functions for these functions. Furthermore, we derive various families of multilinear and multilateral generating functions for extended multivariable hypergeometric functions and their special cases are also given.

The talk is based on collaborations with Professor Esra Erkus-Duman.

[1] M. A. Chaudhry, A. Qadir, H. M. Srivastava and R. B. Paris, *Extended hypergeometric and confluent hypergeometric functions*, Appl. Math. Comput. **159** (2004), 589-602.

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Approximation of solutions to time dependent multidimensional Schrödinger equations

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We present a fast method for high-order approximation of the solution of the Cauchy problem for the multidimensional Schrödinger equation of free particles:

$$i\frac{\partial u}{\partial t} + \Delta_{\mathbf{x}} u = f(\mathbf{x}, t)\,, \qquad u(\mathbf{x}, 0) = g(\mathbf{x}) \tag{1}$$

Here $u = u(\mathbf{x}, t)$ is the wave function depending on the spatial variables $\mathbf{x} = (x_1, ..., x_n) \in \mathbb{R}^n$ and the time $t \in \mathbb{R}$. We suppose that f and g are supported with respect to $\mathbf{x}$ in a hyper-rectangle $[\mathbf{P}, \mathbf{Q}] = \{\mathbf{x} \in \mathbb{R}^n : P_j \leq x_j \leq Q_j, j = 1, ..., n\}$, $f \in C^N([\mathbf{P}, \mathbf{Q}] \times \mathbb{R})$ and $g \in C^N([\mathbf{P}, \mathbf{Q}])$.
Our method consists in approximating the functions f and g via the generating functions introduced in the method of approximate approximations (cf. [1]), which are product of Gaussians and special polynomials. This approach, combined with separated representations of the data, makes our method effective also in high dimensions because only one-dimensional operations are used. Similar to other integral operators of potential theory, we obtain high-order approximations also in higher dimensions up to a small saturation error, which is negligible in computations. We illustrate the efficiency of the method on several examples, up to approximation order 6 and space dimension 200.

The talk is based on a joint work with Vladimir Maz'ya (Linköping University, Sweden) and Gunther Schmidt (WIAS, Berlin, Germany).

[1] V. Maz'ya, G. Schmidt, *Approximate Approximations*, AMS (2007).

Generalized Kantorovich operators on convex compact subsets and their application to evolution problems

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In this talk, we present some recent results concerning a new sequence $(C_n)_{n\geq 1}$ of positive linear operators acting on function spaces defined on a convex compact subset K of some locally convex Hausdorff space.
Their construction involves a Markov operator $T : C(K) \to C(K)$, a real number $a \geq 0$, and a sequence $(\mu_n)_{n\geq 1}$ of probability Borel measures on K. For particular choices of such parameters, operators C_n turn into some known sequence of operators (e.g., Kantorovich operators on the unit interval and the multidimensional hypercube and simplex, together with several of their wide-ranging generalizations scattered in the literature).
We study the approximation properties of C_n's by also stating some estimates of the rate of convergence. Moreover, we discuss some conditions under which operators C_n preserve Lipschitz-continuity and convexity.
We also investigate whether, and for which class of evolution problems, the operators C_n can be useful in approximating the relevant solutions.

[1] F. Altomare, M. Cappelletti Montano, V.L., and I. Raşa., *A generalization of Kantorovich operators for convex compact subsets*, Banach J. Math. Anal., http://dx.doi.org/10.1215/17358787-2010-0008.

[2] F. Altomare, M. Cappelletti Montano, V.L., and I. Raşa., *Elliptic differential operators and positive semigroups associated with generalized Kantorovich operators*, submitted.

Approximation by Jain type Szasz-Durrmeyer operators to Bögel continuous functions

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In this study, we introduce a bivariate generalization of summation-integral type operators based on Lupaş-Jain type bases functions [1,2]. We study its converges properties in terms of modulus of continuity and K-functional and give its degree of approximation. Also, we define a Generalized Boolean Sum (GBS) operators associated with these bivariate Jain type Szasz-Durrmeyer operators and investigate some smoothness properties for these GBS operators. We give some illustrations about convergence of the operators to certain functions by means of graphics.

The talk is based on collaborations with Professor Nurhayat Ispir.

[1] O. Agratini, *On a sequence of linear and positive operators,* Facta Univ. Ser. Math. Inform. **14** (1999), 41-48.

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On the generalized Sylvester polynomials

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In this study, we give new properties of the generalized Sylvester polynomials by using similar approach in [1,2,3]. Various families of multilinear, multilateral generating functions, some special cases and recurrence relations for thqese polynomials are obtained. In addition, we derive a theorem giving certain families of bilateral generating functions for the generalized Sylvester polynomials and the generalized Lauricella functions. Finally, we get several results of this theorem.

The talk is based on collaborations with Professor Esra Erkus-Duman.

[1] S.-J. Liu, S.-D. Lin, H. M. Srivastava and M.-M. Wong, *Bilateral generating functions for the Erkus-Srivastava polynomials and the generalized Lauricella functions*, Appl. Math. Comput. **218** (2012), 7685-7693.

[2] N. Özmen and E. Erkuş-Duman, *Some families of generating functions for the generalized Cesáro polynomials*, J. Comput. Anal. Appl. (accepted for publication).

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Approximation by Positive Linear Operators in Modular Spaces by Power Series Method

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In the present paper, we study the problem of approximation to a function by means of positive linear operators in modular spaces in the sense of power series method. Indeed, in order to get stronger results than the classical cases, we use the power series method which also includes both Abel and Borel methods. An application that satisfies our theorem is also provided.

This work was supported by the Ahi Evran University Scientific Research Projects Coordination Unit. Project Number: FEF.A3.16.033

The talk is based on collaborations with Dr. Tuğba Yurdakadim.

[1] C. Bardaro and I. Mantellini, *Approximation properties in abstract modular spaces for a class of general sampling-type operators,* Appl. Anal. **85** (2006), 383-413.

[2] T. Yurdakadim, *Some Korovkin type results via power series method in modular spaces*, Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat. **65** (2016), 65-76.

Neural network approximation in summation process

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In this paper, we investigate approximation properties of some neural network operators, Cardaliaguet-Euvrard and "squashing" operators, introduced in [1,2] by using Bell's summability process (see [3]). It is well known that the Fourier series of a continous function does not necessarily converge to the function; however its Cesàro summation (also a Bell-type method) does. We discuss a similar approach for these operators.

The talk is based on collaborations with Professor Oktay Duman

[1] G. A. Anastassiou, *Rate of convergence of some neural network operators to the unit-univariate case*, J. Math. Anal. Appl. **212** (1997), 237-262.

[2] G. A. Anastassiou, *Rate of convergence of some neural network operators to the unit-univariate case, revisited*, Mat. Vesnik **65** (2013), 511-518.

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On singular double integrals equipped with summation

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In this study, we prove the pointwise convergence of the two dimensional analogues of the operators studied by Almali [1] at some characteristic points of locally integrable functions. Basically, we follow the lines of Gadjiev [2] and Taberski [3]. Also, we compute the corresponding rate of convergence.

The talk is based on collaborations with Sevgi Esen Almali.

[1] S. E. Almali, *On approximation properties for non-linear integral operators*, arXiv:1702.04190v1 (2017), 1-7.

[2] A. D. Gadjiev, *The order of convergence of singular integrals which depend on two parameters*, Special problems of functional analysis and their applications to the theory of differential equations and the theory of functions, lzdat. Akad. Nauk Azerbaldan, SSR, Baku, (1968), 40-44.

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Approximation properties of Durrmeyer type Bernstein operators based on (p, q)-integers with two variables

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In this paper, the Durrmeyer type Bernstein operators based on (p, q)-integers with two variables are defined. Firstly, few moments of these operators are calculated. Then, we prove Korovkin-type approximation theorems in different function spaces. Moreover, we compute the error of approximation by using modulus of continuity and Lipschitz-type functionals. Additionally, generalization of the Durrmeyer type Bernstein operators based on (p, q)-integers with two variables are given. Finally, numerical results of these operators are presented in detail.

On statistical approximation properties of a generalization of Gamma type operators on (p, q)-integer

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In this presentation, we introduce a genuine type of a (p, q)-generalization of Gamma operators given in [1,2]. After some lemmas and theorems related with test functions and moments of these operators are obtained, we study on the operators in a weighted space of functions and find the rate of these convergences using weighted modulus of continuity. At last, we estimate Voronovskaja type theorem and analyze statistical convergence.

The talk is based on collaborations with Ph.D Student Matanat Gafarli.

[1] H. Karsli, *Rate of convergence of new Gamma type operators for functions with derivatives variation*, Math. Comput. Modelling, **45** (2007) 617-624.

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Generalized Limits in the view of $\mathfrak{B}$-Statistical Convergence

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Let m be the space of bounded real sequences. Banach has shown that there exists positive linear regular functionals on m such that they are invariant under shift operator. It has also been shown that there exists positive linear regular functionals L on m such that $L(\chi_K) = 0$ for every characteristic sequence χ_K of sets, K, of natural density zero. The comparison of such functionals and some applications have been provided. In this paper we define $S_{\mathfrak{B}}$ -limits and $\mathfrak{B}$-Banach limits where $\mathfrak{B}$ is a sequence of infinite matrices. It is clear that if $\mathfrak{B} = (A)$ then these definitions reduce to S_A-limits and A-Banach limits. We also show that the sets of all $S_{\mathfrak{B}}$ -limits and Banach limits are distinct but they have common functionals.

The talk is based on collaborations with Dr. Emre Taş. This paper was supported by the department of Scientific Research Projects of Hitit University. Project No: 19008

[1] A. R. Freedman, *Generalized limits and sequence spaces,* Bull. London Math. Soc. **13** (1981), 224-228.

[2] M. Jerison, *The set of all generalized limits of bounded sequences*, Canad. J. Math. **9** (1957), 79-89.

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Complex analysis and convex optimization and their applications in wave physics

Organizers: S. Nordebo, Y. Ivanenko

Application of perturbation theory to the Frequency Response Analysis of large electrical machines

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The so-called "Frequency Response Analysis" (FRA) is a presently very popular diagnostic method for testing the mechanical integrity of windings in large power transformers, reactors, and rotating electrical machines. In this talk I will use spectral perturbation theory to perform a sensitivity analysis of FRA. I will show that there is a simple relation between changes of the resonant energy associated to different eigenmodes of the system and changes of the characteristic resonant frequencies in the response functions measured within FRA, and I will discuss practical applications of this relation. Furthermore, I will also investigate the usefulness of "time-dependent" perturbation theory (i.e., perturbation theory of the resolvent) in this context.

Absorption and plasmonic resonances for small ellipsoidal particles in lossy media

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A new formula is derived for the absorption cross section of small dielectric ellipsoidal particles embedded in lossy media [1]. The derived expression leads directly to a closed form solution for the optimal conjugate match with respect to the surrounding medium, i.e., the optimal permittivity of the ellipsoidal particle that maximizes the absorption at any given frequency. This defines the optimal plasmonic resonance for the ellipsoid. The conjugate match represents a metamaterial since the corresponding optimal permittivity function can have a negative real part (inductive properties). A necessary and sufficient condition is derived for tuning the Drude model to the optimal conjugate match at a single frequency, and it is found that all the prolate spheroids and some of the (not too flat) oblate spheroids can be tuned into optimal plasmonic resonance at any desired center frequency. Except for the general understanding of plasmonic resonances in lossy media, it is also anticipated that the new results can be useful for feasibility studies with e.g., cancer treatment based on electrophoretic heating in gold nanoparticle suspensions using microwave radiation [2].

The talk is based on collaborations with Sven Nordebo (Linnaeus University, Sweden), Daniel Sjöberg (Lund University, Sweden) and Richard Bayford (Middlesex University, United Kingdom).

[1] M. Dalarsson, S. Nordebo, D. Sjöberg, and R. Bayford, *On the physical limitations for radio frequency absorption in gold nanoparticle suspensions*, arXiv:1703.07605v2 [physics.class-ph] (2017).

[2] E. Sassaroli, K. C. P. Li, and B. E. O'Neil, *Radio frequency absorption in gold nanoparticle suspensions: a phenomenological study*, Journal of Physics D: Applied Physics **45** (2012), 1-15.

Application of Herglotz-Nevanlinna functions to a multiconductor transmission line model based on Green's functions.

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Several Multiconductor Transmission Line models (MTLs) are found in literature. They should be passive in order to have physical meaning because transmission lines are passive circuit elements. However, passivity by construction is not always guaranteed for all models. The authors have recently proposed a Rational Green's-function-based model called Delay-RaG or "DeRaG" where the line delay is explicitly included [1]. The transmission line is described in the Laplace domain by the matrix impedance

$$\mathbf{Z}(s) = \sum_{m=1}^{\hat{m}} \frac{\mathcal{R}_m}{s - p_m} + \underbrace{\mathcal{H}(s)}_{\text{Hyperbolic functions}} \tag{2}$$

where $\mathcal{R}_m$ and p_m are suitable residues and poles computed as described in [1]. The summation takes into account the low frequency behavior of the line, whereas the infinite behavior is retained by virtue of hyperbolic functions that read as coth and csch functions. They also account for the lossless time delay associated with each conductor. In the time domain, it has been proven that the hyperbolic functions read as Dirac Combs. In this paper, we discuss the passivity of the hyperbolic part. In particular, in the Laplace domain, the hyperbolic functions read as Herglotz-Nevanlinna functions, therefore they are positive-real. We conclude that the hyperbolic part of the impedance is passive by construction. Some considerations are made about the positive-realness property of the matrix impedance and its state-space realization.

The talk is based on collaborations with PhD Elena Miroshnikova.

[1] De Lauretis, M., Antonini, G. and Ekman, J., *A delay-rational model of lossy multiconductor transmission lines with frequency-independent per-unit length parameters*, IEEE Transactions on Electromagnetic Compatibility, **57**, 1235–1245, Oct. 2015.

Herglotz functions, sum rules, and fundamental limitations on electromagnetic systems

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An overview of the use of Herglotz functions to derive physical bounds on passive electromagnetic systems is presented. The bounds are derived by identification of a passive system, representation of the system with a Herglotz function, and using integral identities (sum rules) to limit the dynamic response with its low and high-frequency asymptotic expansion. These types of identities and limits are of great interest in many areas of physics and engineering. They also provide insight into the relationship between design parameters. We analyze and present physical bounds for; radar absorbers, temporal dispersion of metamaterials, extraordinary transmission, and antennas. We also compare the theoretical results with state of the art designs.

Passive approximation and optimization with B-splines

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This contribution gives a report on the ongoing research on approximation and optimization of passive electromagnetic systems with Herglotz functions on the real line, B-splines and convex optimization. It is a complement framework to the convex optimization approach for optimal realization of passive structures developed in [1]. The aim of this research is to employ the general B-spline approximation through investigation of the Hilbert transform of B-splines for uniform and non-uniform discretizations to reduce the error of approximation of continuous linear causal system over a finite interval and investigate its realizability. This complement based on B-splines allows also to improve the estimation of physical limitations of passive systems [1, 2] using the sum rule expression

$$\frac{2}{\pi}\int_{0+}^{\infty}\frac{\Im h(\omega)}{\omega^{2n}}\mathrm{d}\omega = a_{2n-1} - b_{2n-1}. \tag{3}$$

Here, $h(\omega)$ is the Herglotz function, $\omega \in \mathbb{R}$ the frequency variable (rad/s), a_{2n-1} and b_{2n-1} are the low- and high-frequency asymptotic parameters of the corresponding Herglotz function, respectively, $n = 1 - N_\infty, ..., N_0$, where N_0 and N_∞ are non-negative integers and where $1 - N_\infty \leq N_0$.
In this presentation, two numerical examples are going to be considered. The first example is related to approximation and passive realization of lossy metamaterials over a given finite frequency bandwidth. In the second example, passive approximation and optimization are used to identify a dispersion model that fits the dielectric response of high-voltage insulation material.

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[2] Mats Gustafsson and Daniel Sjöberg, *Sum rules and physical bounds on passive metamaterials*, New Journal of Physics **12** (2010), 043046.

Moment problems and optimization: A global-analysis approach to problems with rationality constraints

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Moment problems are ubiquitous throughout engineering and science and have had a profound impact on the development of modern mathematical analysis. Power moments of probability measures play an important role in statistical modeling and in its application to information theory, communications, signals and systems. Applications of the trigonometric moment problem to systems and control also have a long and fruitful history, including the rational covariance extension problem. Analytic interpolation problems are an important class of moment problems with applications to circuit theory, power systems, robust control, signal processing, spectral estimation, model reduction, and stochastic realization theory. A common feature arising in many engineering applications is that the desired solution needs to be a rational positive measure of a bounded degree, reflecting its implementability by a finite dimensional device. These moment problems are typically underdetermined and give rise to families of particular solutions, and finding a solution that also satisfies a natural optimality criterion or design specification is an important general problem. While this nonclassical version of the moment problem is decidedly nonlinear, there exists a natural, universal family of strictly convex optimization criteria defined on the convex set of particular solutions. Taking a global-analysis approach, where one studies the family of solutions as a whole, provides a powerful paradigm for smoothly parameterizing, comparing and shaping the solutions based on various additional design criteria. It also enables us to establish the smooth dependence of solutions on problem data, thereby facilitating tuning of solutions. It also provides a generalization of the generalized interpolation results of Sarason.

Fundamental bound on extraordinary transmission through periodically perforated screens

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Fundamental bound for bandwidth of transmission through perforated metal screens is constituted in the form of an integral identity and validated by measurements and numerical simulations. We consider a transmission of a planar electromagnetic wave through a thin metal screen with periodic perforations. The fundamental bound imposes that the total transmission bandwidth is proportional to a static polarizability of the screen. Using this we show that *extraordinary transmission* (i.e. transmission greater than the non-metal fraction of the screen area) is present in particular frequency bands for any periodically perforated screen. We also illustrate how the bound facilitates design and optimization of frequency selective arrays.

The talk is based on collaborations with Johan Lundgren, Casimir Ehrenborg, Mats Gustafsson, Daniel Sjöberg, Andreas Ericsson (Lund University), Yevhen Ivanenko (Linnæus University) and B.L.G. Jonsson (KTH Royal Insitute of Technology).

An integral representation for Herglotz-Nevanlinna functions in several variables and its consequences

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Herglotz-Nevanlinna functions are holomorphic functions defined in the poly-upper half-plane having non-negative imaginary part. When considering the case of one complex variable, this is an old topic that has first been considered by Herglotz, Nevanlinna, Pick and others around 100 years ago. Partial results for the several variable case have been known since the 1970s, but no complete characterization of thic class of functions has been given before now.
In this talk, we will first present a complete characterization of the class of Herglotz-Nevanlinna functions in several variables via an integral representation involving a real constant, a liner part, a given integral kernel and a positive Borel measure. Afterwards, consequences of this integral representations will be investigated, focusing mainly on the class of boundary measures of Herglotz-Nevanlinna functions.

The talk is based on collaborations with Annemarie Luger.

[1] Annemarie Luger, Mitja Nedic, *A characterization of Herglotz-Nevanlinna functions in two variables via integral representations*, arXiv: 1605. 06232v2.

[2] Annemarie Luger, Mitja Nedic, *An integral representation for Herglotz-Nevanlinna functions in several variables*, Research reports in mathematics, 2017-1.

Ultradistributions and generalized Herglotz-Nevanlinna functions

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Herglotz-Nevanlinna functions map analytically the open upper complex half-plane to the closed one. They represent naturally passive time-translationally invariant linear systems. Generalizations of these functions are investigated in the talk as representations of causal rather than passive systems recalling that passivity implies causality. A basis for the investigation is the definition [1,2] stating that an ultradistribution $u(t)$ is said to be zero in the Jones' sense for $t < a$, if for some real d, $u(t) = v(t + \mathrm{i}d)$, where $v(t)$ is a tempered distribution with $v(t) = 0$ for $t < a$. The characterization [1] of the Fourier-Laplace transform $U(\omega)$ of $u(t)$ is reviewed and discussed as a generalization of Herglotz-Nevanlinna functions. $U(\omega)$ may have exponential growth modelling unstable systems. A further generalization of causality in the frequency domain is proposed to cover the case when stable and unstable waves interact.

The talk is based on collaborations with Professors Sven Nordebo and Joachim Toft.

[1] D. S. Jones, J. D. Morgan, *A linear model of a finite amplitude Helmholtz instability*, Proceedings of the Royal Society of London: A Mathematical, Physical and Engineering Sciences **338**(1612), (1974), 17-41.

[2] D. S. Jones, *The theory of generalized functions*, Cambridge Univ. Press (1982).

Semi-infinite moment constraints for Stieltjes functions with applications in homogenization theory

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Consider the following Stieltjes function [1] representation

$$p(s) = 1 + \int_0^1 \frac{\mathrm{d}\mu(\xi)}{\xi - s}, \tag{4}$$

where $\mathrm{d}\mu(\xi)$ is a finite positive Borel measure and $s \in \mathbb{C}$ a parameter, $s \notin [0, 1]$. A typical application is with the homogenization of a random media consisting of two phases with dielectric constants ϵ_1 and ϵ_2, and where $p(s) = \epsilon^*/\epsilon_1$ denotes the effective dielectric parameter, and $s = 1/(1 - h)$ where $h = \epsilon_2/\epsilon_1$, see [2]. We consider the bounds on $p(s)$ subjected to the moment constraints

$$\int_0^1 \xi^i \mathrm{d}\mu(\xi) = \mu^{(i)}, \qquad i = 0, \ldots, m - 1, \tag{5}$$

where $\mu^{(i)}$ are given, and we study their interpretation in the context of semi-infinite linear programming [3]. In particular, Carathéodory's theorem [3] provides a prominent dimensionality result regarding the generation of convex cones in $\mathbb{R}^n$, that facilitate a finite-dimensional implementation of the constraints in (5). In this contribution, we review the mathematical basis of semi-infinite linear programming and discuss its applicability in homogenization theory.

[1] I. S. Kac and M. G. Krein. *R-functions - Analytic functions mapping the upper halfplane into itself.* Am. Math. Soc. Transl., **103**(2) (1974), 1–18.

[2] K. Golden and G. Papanicolaou. *Bounds for effective parameters of heterogeneous media by analytic continuation.* Communications in Mathematical Physics, **90** (1983), 473–491.

[3] E. J. Anderson and P. Nash. *Linear programming in infinite-dimensional spaces.* John Wiley & Sons, Ltd. (1987).

Physical Bounds and Convex Optimization Approximations for Electromagnetically Functional Surfaces

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An electromagnetically functional surface is typically realized as a periodic structure in the xy-plane with finite extent in the z-direction, providing functionality such as absorption, frequency or polarization selectivity, artificial magnetic conductivity etc. By controlling the microstructure or the temporal dispersion of the materials, the electromagnetic functionality can be controlled. The reflection or transmission coefficient of the structure can be associated with a Herglotz function, whose asymptotic values in the low and high frequency limit represent physical constraints such as the total thickness of the structure. Using the analyticity of Herglotz functions, sum rules and physical bounds restricting the electromagnetic functionality in terms of asymptotic values can be derived under some restrictions, which can be relaxed by considering convex optimization approximations based on an integral representation of Herglotz functions. The results can be used to characterize the maximum bandwidth obtainable for a certain thickness of the structure.

The talk is based on collaborations with Mats Gustafsson and Sven Nordebo.

Complex and functional analytic methods for differential equations

Organizers: H. Begehr, O. Celebi, J. Y. Du

On a $\psi-$weighted Cauchy-Riemann type operator in the complex plane

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In this talk we introduce a non-constant weighted Cauchy-Riemann type operator in the complex plane, where the weights depend on the complex variable z. We construct a fundamental solution for this operator inspired by the idea for the construction of the Levi function proposed by Miranda in the Clifford algebras framework. Therefore, a Cauchy-Pompeiu type integral representation formula is also discussed. We present some examples of this modified Cauchy-Riemann operator obtained with different weights, in particular, we exhibit the fact that this operator is a generalization of the classical Cauchy-Riemann Operator in the complex plane.

The talk is based on collaborations with Antonio Di Teodoro and Zuly Salinas.

[1] H. Begehr, *Boundary value problems in complex analysis I*, Boletín de la Asociación Matemática Venezolana **1** (2005), 65–85.

[2] C. Miranda, *Partial differential equations of elliptic type*, Springer-Verlag, Berlin, 1970.

[3] W. Tutschke and C. J. Vanegas, *Métodos del análisis complejo en dimensiones superiores*, Ediciones IVIC, Caracas, 2008.

Robin problem for Poisson equation

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The two basic boundary value problems for the Poisson equation are the Dirichlet and the Neumann problems. Their related fundamental solutions are the harmonic Green $G_1(z,\zeta)$ and Neumann $N_1(z,\zeta)$ functions. A linear combination of these two boundary value problems is the Robin problem. The related fundamental solution, the modified Robin function $R_{1;\alpha,\beta}(\zeta,z)$, can be chosen as an interpolation between the Green and the Neumann functions. For plane domains this is done in [1] with complex notation. While the Dirichlet problem is unconditionally solvable this is in general not the case for the other ones. In explicit form, however, this was shown only for particular domains like the unit disc. In [2] the solvability condition is given for any admissible plane domain. However, the formulation of the Robin boundary condition in a proper way is not so obvious as one might believe when looking at the case of the unit disc. As an example an elliptic ring domain, bounded by two confocal ellipses, serves, see [3].

The talk is based on collaboration with Saule Burgumbayeva and Bibinur Shupeyeva.

[1] H. Begehr, T. Vaitekhovich, *Modified harmonic Robin functions.* Complex Var. Elliptic Eqs. 58 (2013), 483–496.

[2] H. Begehr, S. Burgumbayeva, B. Shupeyeva. *Remark on Robin problem for Poisson equation.* Complex Var. Elliptic Eqs., to appear, doi.org/10.1080/17476933.2017.1303052.

[3] H. Begehr. *Fundamental solutions to the Laplacian in plane domains bounded by ellipses.* In: Mathematics and Computing, 3rd Intern. Conf., ICMC 2017, Haldia, India, January 2017, Proc., D. Giri, R.N. Mohapatra, H. Begehr, M.S. Obaidat (eds.). Communications in Computer and Information Science 655, Springer, Singapore, 2017, 293–311.

The problem of integral geometry of Volterra type

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We study a new problem of reconstruction of a function in a strip from their given integrals with known weight function along polygonal lines. We obtain two simple inversion formulas for the solution to the problem. We prove uniqueness and existence theorems for solutions and obtain stability estimates of a solution to the problem in Sobolevs spaces and thus show their weak ill-posedness. Then we consider integral geometry problems with perturbation. The uniqueness theorems are proved and stability estimates of solutions in Sobolev spaces are obtained. The given results are continuation of our research [1-5].

[1] Akram Kh. Begmatov, *Weakly ill-posed Volterra-type problems of integral geometry*, Doklady Mathematics **54:1** (1996), 536-537.

[2] Akram Kh. Begmatov, *Problems of integral geometry for special curves and surfaces with singularities in vertices*, Doklady Mathematics **57:1** (1998), 117-118.

[3] Akram Kh. Begmatov, *Two new classes of problems in integral geometry*, Doklady Mathematics **57:3** (1998), 427-429.

[4] Akram Kh. Begmatov, Z. Kh. Ochilov, *Integral geometry problem with a discontinuous weight function*, Doklady Mathematics **80:3** (2009), 823-825.

[5] Akram. H. Begmatov, M. E. Muminov, Z. KH. Ochilov, *The problem of integral geometry of Volterra type with a weight function of a special type*, Journal Mathematics and Statistics **3(5)** (2015), 113-120.

Dirichlet type boundary value problems in polydiscs

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In this presentation, we consider the Riquier problem for polyharmonic equations defined in the unit polydisc of $\mathbb{C}^n$. This is a Dirichlet type boundary value problem for higher order linear complex differential equations with a polyharmonic leading term. After deriving a Green's function, we present the solution for a model equation with homogeneous boundary conditions. Afterwards we have converted the differential equation into a singular integral equation and employed Fredholm theory to obtain the solution of the linear equation for Riquier boundary value problem on a unit polydisc in $\mathbb{C}^2$.

Some applications of the theory of conjugate differential forms

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Two (homogeneous) differential forms u and v are said to be conjugate if $du = \delta v$, $\delta u = 0$, $dv = 0$, where d and δ denote the differential and the co-differential operators respectively.
Conjugate forms extend to n real variables the concept of holomorphic function of one complex variable. Moreover, solutions of the Moisil-Theodorescu system and Fueter system can be considered as very particular cases of conjugate differential forms in $\mathbb{R}^3$ and in $\mathbb{R}^4$, respectively; an example in $\mathbb{R}^n$ is given by harmonic vectors, i.e. vectors $(w_1, \ldots, w_n)$ such that $\mathrm{div}(w_1, \ldots, w_n) = 0$ and $\mathrm{curl}(w_1, \ldots, w_n) = 0$ or, more generally, closed and co-closed differential forms.
The aim of the present talk is to survey recent results we have obtained for conjugate differential forms and - more generally - for self-conjugate differential forms (i.e. non-homogeneous differential forms U such that $dU = \delta U$). In particular applications of the theory of such forms to Laplace series and to some boundary value problems will be presented.

A Bitsadze-Samarsky boundary condition for the elliptic-parabolic volume potential

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In this talk an integral representation of elliptic-parabolic volume potential is discussed, and then using boundary conditions of heat and Newtonian potentials we find the Bitsadze-Samarsky boundary condition for the considered volume potential.

The talk is partially based on ideas of the papers [1] and [2].

[1] T.Sh. Kal'menov and M. Otelbaev. *Boundary criterion for integral operators.* Dok. Math., **93** (2016), 58-61.
[2] T. Sh. Kal'menov and D. Suragan. *To spectral problems for the volume potential* , Dok. Math., **80** (2009), 646-649.

Certain boundary-value problems for Beltrami equations on non-rectifiable curves

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We solve Riemann type boundary-value problem for the equation

$$\frac{\partial \phi}{\partial \bar{z}} = \mu \frac{\partial \phi}{\partial z},$$

where the coefficient μ is defined in the whole complex plane, $|\mu(z)| \leq \beta < 1$, and ϕ is the desired function. Let Γ be a simple closed curve (generally speaking, non-rectifiable) dividing the complex plane onto domains D^+ and D^-, $\infty \in D^-$. We seek this equation in $\mathbb{C} \setminus \Gamma$ satisfying functions ϕ such that they have limit values $\phi^{\pm}(t)$ at any point $t \in \Gamma$ from D^+ and from D^- respectively, and

$$\phi^+(t) = G(t)\phi^-(t) + g(t), \quad t \in \Gamma.$$

Here $G(t)$ and $g(t)$ are given functions.
We solve the problem in terms of a generalization of curvilinear integral for non-rectifiable paths and so called Marcinkiewicz exponents; these characteristics of non-rectifiable curves are introduced in papers [1, 2]. As a result, we obtain solvability conditions for the problem.

The talk is based on collaborations with David B. Katz.

[1] David B. Katz, *New metric characteristics of non-rectifiable curves with applications*, Siberian Mathematical Journal **57** (2016), 364-372.

[2] Boris A. Kats, David B. Katz, *Marcinkiewicz exponents and integrals over non-rectifiable paths*, Mathematical Methods in the Applied Sciences **39** (2016), 3402-3410.

On one interpretation of a uniqueness theorem of the theory of analytic functions

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The present work deals with the interpretation of a uniqueness theorem of the theory of analytic functions, its generalization and some applications.

The talk is based on collaborations with G. Akhalaia, G. Makatsaria, and T. Vekua (Tbilisi).

Regularity and controllability of the heat equation with memory and loaded masses on graphs

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Recent interest in control and inverse problems for differential equations on graphs is motivated by applications to important problems of classical and quantum physics, chemistry, biology, and engineering.
In this talk we describe regularity and exact boundary controllability results for the heat equation with memory on compact graphs. The heat equation with memory is defined on each edge of the graph and point masses attached to the interior vertices. Dirichlet or Neumann type controls are applied to the boundary vertices. We demonstrate that the wave transmitted through the mass is more regular than the incident wave. We prove the sharp regularity results for our initial boundary value problems in non-symmetric Sobolev spaces with index depending on the combinatorial distance from the boundary. The exact controllability results are proved for graphs without cycles (trees) and controls applied to all or all but one boundary vertices.

The talk is based on collaborations with Sergei Avdonin (Univ. Alaska Fairbanks).

[1] Sergei Avdonin, Gulden Murzabekova, Karlygash Nurtazina, *Source identification for the heat equation with memory*, University of Minnesota, Institute for Mathematics and its Application, **2459** (2015), 1-8.

Boundary Inverse Problems for the Heat Equation with Memory and Loaded Masses on Graphs

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In this talk we describe identifiability results for the heat equation with memory on compact graphs without cycles (trees). The heat equation with memory with a source-like term is defined on each edge of the graph, and point masses attached to the interior vertices. We define the response operator R^T, i.e. the dynamical Dirichlet-to-Neumann map associated with the set of all or all but one boundary vertices. We demonstrate that the operator R^T known for T greater than the exact controllability time of the system uniquely determines the sources, lengths of the edges, attached masses, and the topology of the graph.
The proofs are based on the boundary control method and the leaf peeling method for the heat equation with memory developed in our previous papers.

The talk is based on collaborations with Sergei Avdonin (Univ. Alaska Fairbanks).

[1] Sergei Avdonin, Gulden Murzabekova, Karlygash Nurtazina, *Source identification for the heat equation with memory*, University of Minnesota, Institute for Mathematics and its Application, **2459** (2015), 1-8.

On maximal regularity of differential and difference operators

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In the space $L_2(-\infty, \infty)$, we discuss the maximal regularity (separability) of the following minimal closed second-order differential operator with unbounded coefficients:

$$Ly = -y'' + r(x)y' + q(x)y$$

as well as its difference analogue l. Since the term $r\frac{dy}{dx}$ may not be a small perturbation due to the growth of r at infinity, the properties of L are very different from the properties of the Sturm-Liouville operator (in the case $r = 0$).
We show that the difference operator l is maximally regular if the coefficient r satisfies certain natural conditions, and q obeys r in terms of the growth rate. But, the maximal regularity estimate depends on the step of difference. If the step of the difference is directed to zero, then the maximal regularity estimate does not hold. We also show that under natural conditions on the coefficients the maximal regularity estimate holds for a wide class of second order difference operators.

This research was supported by the grant 5132/GF4 and the target program 0085/PTSF-14 of the Ministry of Science and Education of the Republic of Kazakhstan.

On a generalised Samarskii–Ionkin type problem for the Poisson equation

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The Dirichlet and Neumann boundary value problems play a key role in the theory of harmonic functions. Another important problem, called periodic boundary value problem, arises when one considers the problem in a one-dimensional case or in a multidimensional parallelepiped. For the first time, in [1, 2], a new class of boundary value problems for the Poisson equation in a multidimensional ball $\Omega = \{x \in \mathbb{R}^n : \ |x| < 1\}$ was introduced. These problems are analogous to the classical periodic boundary value problems.

If we turn to the non-classical problems, then one of the most popular problems is the Samarskii–Ionkin problem, arisen in connection with the study of the processes occurring in the plasma in the 70s of the last century by physicists.

In this talk an analog of the Samarskii–Ionkin type boundary value problem for the Poisson equation in the multidimensional ball is considered:

$$-\Delta u = f(x), \ x \in \Omega,$$
$$u(x) + \alpha u(x^*) = \tau(x), \ \frac{\partial u}{\partial r}(x) - \frac{\partial u}{\partial r}(x^*) = \nu(x), \ x \in \partial\Omega_+.$$

Here as usual $\partial\Omega_+$ is a part of the sphere $\partial\Omega$, for which $x_1 \geq 0$; each point $x = (x_1, x_2, ..., x_n) \in \Omega$ is matched by its "opposite" point $x^* = (-x_1, \alpha_2 x_2, ..., \alpha_n x_n) \in \Omega$, where the indices $\alpha_j, j = 2, \ldots, n$ take one of the values ± 1. Clearly, if $x \in \partial\Omega_+$, then $x^* \in \partial\Omega_-$.

[1] M. A. Sadybekov, and B. Kh. Turmetov. *On analogues of periodic boundary value problems for the Laplace operator in a ball*, Eurasian Mathematical Journal, **3**:1 (2012), 143–146.

[2] M. A. Sadybekov, and B. Kh. Turmetov. *On an analog of periodic boundary value problems for the Poisson equation in the disk*, Differential Equations. **50**:2 (2014), 268–273.

Complex-analytic and Wiener-Hopf methods in the applied sciences

Organizers: G. Mishuris, S. Rogosin

On the accuracy of the numerical solution of the boundary integral equation with the generalized Neumann kernel

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Recently, a fast numerical method has been presented in [1] for solving the boundary integral equation with the generalized Neumann kernel for multiply connected domains of connectivity $m+1$ in $O((m+1)n\ln n)$ operations where n is the number of nodes in the discretization of each boundary component. In this talk, we study the accuracy of the numerical solution of the integral equation. Further, we compare the accuracy of the integral equation to other well known integral equations such as the Mikhlin integral equation, the Kerzman-Stein-Trummer integral equation for the Szegö kernel, and the Razali-Nashed-Murid integral equation for the Bergman kernel. We discuss several explicit examples of popular test domains and some general estimates.
The presented numerical results illustrate the advantages of the boundary integral equation with the generalized Neumann kernel which gives accurate results even for domains of very high connectivity, domains with piecewise smooth boundaries, domains with close-to-touching boundaries, and domains with complex boundary.

[1] M.M.S. Nasser, *Fast solution of boundary integral equations with the generalized Neumann kernel*, Electron. Trans. Numer. Anal. **44** (2015), 189–229.

Approximate factorization of a class of matrix functions

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In our talk we describe a recently proposed method for approximate factorization of a class of continuous (non-rational) matrix-functions $G \in (\mathcal{C}(\mathbb{R}))^{n\times n}$, i.e. their representation in the form

$$G(x) = G^-(x)\Lambda(x)G^+(x),$$

with continuous invertible factors $G^\pm(x), (G^\pm)^{-1}(x)$, possessing analytic continuation into the corresponding half-plane $\Pi^\pm = \{z = x+iy : \text{Im } \pm z > 0\}$, and

$$\Lambda(x) = \text{diag}\left(\left(\frac{x-i}{x+i}\right)^{\varkappa_1}, \dots, \left(\frac{x-i}{x+i}\right)^{\varkappa_n}\right),\ \varkappa_1,\dots,\varkappa_n \in \mathbb{Z}.$$

The essential property of the considered matrices is that they become close (after suitable transformation) to a unit matrix at infinity. The class of matrices being considered is motivated by their applicability to various problems. The properties and steps of the asymptotic procedure are discussed in detail.
The main attention is paid to the following cases: (a) canonical factorization when the partial indices $\varkappa_j$ are equal to zero (see [1]); (b) factorization with stable partial indices, i.e. max $\varkappa_j -$ min $\varkappa_{\le} 1$ (see [2]); (c) factorization with unstable partial indices, i.e. max $\varkappa_j -$ min $\varkappa_{>} 1$ (see [3]).

The talk is based on collaborations with Gennady Mishuris (Aberystwyth University, UK). It is supported by the People Programme of the EU 7th Programme FP7/2007- 2013/ under REA grant agreement PIRSES-GA-2013- 610547 - TAMER and by Belarusian Fund for Fundamental Scientific Research, grant 17MS-002.

[1] Gennady Mishuris, Sergei Rogosin, *An asymptotic method of factorization of a class of matrix functions*, Proc. Royal Soc. A **470** (2014), No. 20140109.

[2] Gennady Mishuris, Sergei Rogosin, *Factorization of a class of matrix-functions with stable partial indices*, Mathematical Methods in the Applied Sciences **39** (2016), 3791-3807.

[3] Gennady Mishuris, Sergei Rogosin, *Regular approximate factorization of a class of matrix-function with an unstable set of partial indices*, http://arxiv.org/abs/1704.07374.

On the Toeplitz plus Hankel integral equation

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The integral equation with the Toeplitz plus Hankel kernel is of the form

$$f(x) + \int_0^\infty [k_1(x+y) + k_2(x-y)]f(y)dy = g(x), \quad x > 0, \tag{6}$$

here g, k_1, k_2 are given, and f is an unknown function. This equation has many useful applications. However, this integral equation can be solved in closed form only in some particular cases of the Hankel kernel k_1 and the Toeplitz kernel k_2. The solution of equation (6) in closed form in the general case is still open. In several known particular cases of the equation (6) the Toeplitz kernel k_2 is an even function, and the equation (6) takes the form

$$f(x) + \int_0^\infty [k_1(x+y) + k_2(|x-y|)]f(y)dy = g(x), \quad x > 0. \tag{7}$$

In this talk we give several particular cases of the equation (7), when the equation can be solved in the closed form. We consider also the Toeplitz plus Hankel integral equation (6), for the case $k_1 = k_2$, but without the restriction that k_2 is an even function, and also the equation being given on the whole real line

$$f(x) + \int_0^\infty [k(x+y) + k(x-y)]f(y)dy = g(x), \quad -\infty < x < \infty. \tag{8}$$

The tools to solve these equations in closed form are the Fourier-cosine and Hartley convolutions.

The talk is based on collaborations with Nguyen Xuan Thao, Nguyen Thanh Hong, and Hoang Thi Van Anh.

Special interest group: IGCVPT Complex variables and potential theory

Organizers: T. A. Azeroglu, A. Golberg, M. L. de Cristoforis, S. Plaksa

The Dirichlet problem and boundary regularity for p-harmonic functions with respect to the Mazurkiewicz boundary

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A *p-harmonic function* on unweighted $\mathbf{R}^n$ is a continuous weak solution of the p-Laplace equation

$$\Delta_p u := \operatorname{div}(|\nabla u|^{p-2}\nabla u) = 0.$$

The results discussed in this talk hold on metric spaces, but most of the interesting questions already appear on unweighted $\mathbf{R}^n$.
Consider a bounded domain G in $\mathbf{R}^n$ and let $f \in C(\partial G)$. In the Dirichlet problem we seek a solution which is p-harmonic in G and which "attains" the given boundary data f at the boundary. In general we cannot require continuity up to the boundary (not even for harmonic functions), but it turns out that the Perron method is well suited for solving the Dirichlet problem.
A natural question is when the Perron solution Pf attains the given boundary values as limits: A boundary point $x_0 \in \partial G$ is *regular* if

$$\lim_{G \ni y \to x_0} Pf(y) = f(x_0) \quad \text{for all } f \in C(\partial G).$$

In some situations, such as the slit disc

$$G = \{x = (x_1, x_2) : |x| < 1\} \setminus ([0,1) \times \{0\}),$$

it can be natural to have different boundary conditions from the top and from the bottom at the points on the slit. In more complicated situations it is far from clear how to define such a generalized boundary.
I will discuss one such approach namely the Mazurkiewicz boundary (which is also related to prime ends), and how the corresponding theory with respect to this boundary looks like.

The talk is based on collaborations with Jana Björn (Linköping), Nageswari Shanmugalingam (Cincinnati) and Tomas Sjödin (Linköping).

Fixed points of holomorphic mappings

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Let $\mathbb{B}$ be an open unit ball in a complex Banach space X. In this talk we discuss conditions that ensure that the set $\mathbb{B} \cap F(\mathbb{B})$ contains the fixed point set of F. A simplest sufficient condition for $F \in \operatorname{Hol}(\mathbb{B}, X)$ to have an interior fixed point in $\mathbb{B}$ consists of the invariance condition

$$F(\mathcal{D}) \subset \mathcal{D} \quad \text{and} \quad \sup_{x \in \mathcal{D}} \left\| F'(x) \right\| < 1 \tag{9}$$

for some closed convex subset $\mathcal{D} \subset \mathbb{B}$. However, condition (9) is too strong and constrains us to use them for solving other problems.
A breakthrough in this direction was done by Earle and Hamilton [1]. However (even in the one-dimensional case) the assumptions of the Earle-Hamilton Theorem become not applicable if F does not map the open unit ball $\mathbb{B}$ into itself. Moreover, many examples show that F might be unbounded on $\mathbb{B}$ but still have a unique fixed point inside.
The goal of our work is to assign such conditions that provide the existence and uniqueness of the fixed point in the open unit ball $\mathbb{B}$ of a general Banach space X for a mapping $F \in \operatorname{Hol}(\mathbb{B}, X)$, which even is not necessarily bounded on $\mathbb{B}$.

The talk is based on collaborations with David Shoikhet.

[1] C. J. Earle and R. S. Hamilton, *A fixed point theorem for holomorphic mappings*, Proc. Symp. Pure Math. **16**, Amer. Math. Soc., Providence, R.I., (1970), 61–65.

Regularity of mappings with integrally restricted moduli

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We consider a class of homeomorphisms of domains in $\mathbb{R}^n$ with integrally bounded from above p-moduli of the families of curves, which essentially extends the well-known classes of mappings such as quasiconformal, quaiisometric, Lipschitzian, etc.
In the talk we survey the known results in this field, but mainly establish new differential features of such mappings. A group of open related problems will be also presented.

The talk is based on collaborations with Ruslan Salimov, Institute of Mathematics of National Academy of Sciences, Kiev, Ukraine.

[1] Anatoly Golberg, *Differential properties of* (α, Q)*-homeomorphisms*, Further progress in analysis, 218–228, World Sci. Publ., Hackensack, NJ, 2009.

[2] Anatoly Golberg, Ruslan Salimov, *Mappings with upper bounds p-moduli*, Contemp. Math., **659** (2016), 91–113.

Loewner theory in the study of biholomorphic mappings with A-parametric representation on the unit ball in $\mathbb{C}^n$

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In this talk we survey recent results in the theory of Loewner chains and the generalized Loewner differential equation on the unit ball $\mathbb{B}^n$ in $\mathbb{C}^n$. A key role in our discussion is played by the Herglotz vector fields associated with the generalized Loewner differential equation on $\mathbb{B}^n$. We point out main differences between the one variable theory and that in higher dimensions. A special interest in this talk consists in various extremal problems associated with the compact family $S_A^0(\mathbb{B}^n)$ of normalized biholomorphic mappings which have A-parametric representation on $\mathbb{B}^n$, where $A \in L(\mathbb{C}^n)$ such that $k_+(A) < 2m(A)$, $k_+(A)$ is the Lyapunov index of A and $m(A) = \min_{\|z\|=1} \Re\langle A(z), z\rangle$. Particular cases, open problems, and questions will be also mentioned.

The talk is based on collaborations with Ian Graham (Toronto), Hidetaka Hamada (Fukuoka), and Mirela Kohr (Cluj-Napoca)

[1] I. Graham, H. Hamada, G. Kohr, M. Kohr, *Extremal properties associated with univalent subordination chains in* $\mathbb{C}^n$, Math. Ann., **359** (2014), 61–99.
[2] I. Graham, H. Hamada, G. Kohr, M. Kohr, *Support points and extreme points for mappings with A-parametric representation in* $\mathbb{C}^n$, J. Geom. Anal., **26** (2016), 1560–1595.

G-monogenic mappings in the algebra of complex quaternions

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Let $\mathbb{H}(\mathbb{C})$ be the quaternion algebra over the field of complex numbers, whose basis consists of the elements $1, I, J, K$. In the algebra $\mathbb{H}(\mathbb{C})$ there exists another basis $\{e_1, e_2, e_3, e_4\}$ such that multiplication table in a new basis can be represented as

$\cdot$	e_1	e_2	e_3	e_4
e_1	e_1	0	e_3	0
e_2	0	e_2	0	e_4
e_3	0	e_3	0	e_1
e_4	e_4	0	e_2	0

.

The unit of the algebra can be decomposed as $1 = e_1 + e_2$.
Let $i_1 = 1, i_2, i_3$ be elements of $\mathbb{H}(\mathbb{C})$ which are linearly independent over the field of real numbers. We consider a new class of quaternionic mappings, so-called, G-monogenic (i. e. continuous and differentiable in the sense of Gateaux) mappings of the variable $xi_1 + yi_2 + zi_3$, where x, y, z are real.
We obtain a constructive description of all mentioned mappings by means of holomorphic functions of complex variables. It follows from this description that G-monogenic mappings have Gateaux derivatives of all orders.

A space of kernels for volume potentials

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The analysis of the dependence of integral operators upon perturbations plays an important role in the analysis of perturbed boundary value problems. In this talk we focus on the mapping properties of the volume potentials with weakly singular kernels. We introduce a special weighted class of kernels, which are analytic away from the singularities, and we prove that the map which takes a density function and a kernel to a (suitable restriction of the) volume potential is bilinear and continuous with values in a Roumieu class of analytic functions. We consider both the periodic and the nonperiodic case. Then we show some applications.

The talk is based on joint work with Matteo Dalla Riva, The University of Tulsa, Oklahoma, USA, and Paolo Musolino, Aberystwyth University, UK.

Some regularizing properties of the double layer heat potential

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In this talk we consider the integral operator associated to the double layer heat potential defined on the boundary of an infinite parabolic cylinder and we prove some regularizing properties in parabolic Schauder spaces. We also introduce ad hoc norms for kernels of integral operators on parabolic cylinders in order to prove continuity results of integral operators upon variation of the kernel, which we apply to the layer heat potentials. Some of our results can be found in [1].

The talk is based on collaborations with Massimo Lanza de Cristoforis (Padova).

[1] M. Lanza de Cristoforis, P. Luzzini, *Time dependent boundary norms for kernels and regularizing properties of the double layer heat potential*, to appear in Eurasian Math. J., 2017.

Monogenic functions in biharmonic boundary value problems

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We consider a commutative algebra $\mathbb{B}$ over the field of complex numbers with a basis $\{e_1, e_2\}$ satisfying the conditions $(e_1^2 + e_2^2)^2 = 0$, $e_1^2 + e_2^2 \neq 0$. Let D be a bounded simply connected domain in the Cartesian plane xOy and $D_\zeta = \{xe_1 + ye_2 : (x, y) \in D\}$. Components of every monogenic function

$$\Phi(xe_1 + ye_2) = U_1(x, y)\, e_1 + U_2(x, y)\, ie_1 + U_3(x, y)\, e_2 + U_4(x, y)\, ie_2$$

having the classic derivative in D_ζ are biharmonic functions in D, i.e., $\Delta^2 U_j(x, y) = 0$ for $j = 1, 2, 3, 4$.
We consider a Schwarz-type boundary value problem: to find a continuous function $\Phi : \overline{D_\zeta} \longrightarrow \mathbb{B}$ which is monogenic in a domain D_ζ when values of components U_1, U_3 are given on the boundary ∂D_ζ:

$$U_1(x, y) = u_1(\zeta)\,, \quad U_3(x, y) = u_3(\zeta) \qquad \forall\, \zeta = xe_1 + ye_2 \in \partial D_\zeta.$$

This problem is associated with the following problem: to find a biharmonic function $V(x, y)$ in D when boundary values of its partial derivatives $\partial V/\partial x$, $\partial V/\partial y$ are given on the boundary ∂D. The problem is also associated with the *principal biharmonic problem*: to find a biharmonic function $V(x, y)$ in D, which is continuously extended together with partial derivatives of the first order up to the boundary ∂D, when its values and values of its outward normal derivative are given on ∂D.
Using a hypercomplex analog of the Cauchy type integral, we reduce the mentioned Schwarz-type boundary value problem to a system of integral equations and establish sufficient conditions under which this system has the Fredholm property. For a half-plane and for a disk, solutions are obtained in explicit forms by means of Schwartz-type integrals.

The talk is based on collaborations with Dr. Serhii Gryshchuk.

Series expansion for the effective conductivity of a non-ideal dilute composite

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This talk is devoted to a singular perturbed non-ideal transmission problem in a dilute two-phase composite with thermal resistance at the interface. The composite is obtained by introducing into an infinite homogeneous planar matrix a periodic set of inclusions of a different material. The diameter of each inclusion is proportional to a real positive parameter ϵ. Dalla Riva and Musolino have shown in [1] that under suitable assumptions the effective thermal conductivity can be expressed by means of a real analytic function of the singular perturbation parameter ϵ and accordingly can be represented as a convergent power series of ϵ. In this talk we will show a constructive method to compute explicitly the coefficients of such series by solving recursive systems of integral equations.

The talk is based on joint work with M. Dalla Riva and P. Musolino.

[1] M. Dalla Riva, P. Musolino, *A singularly perturbed nonideal transmission problem and application to the effective conductivity of a periodic composite*, SIAM J. Appl. Math., **73** (2013), 24-46.

[2] M. Dalla Riva, P. Musolino, S. V. Rogosin, *Series expansions for the solution of the Dirichlet problem in a planar domain with a small hole*, ASYMPTOTIC ANALYSIS, **92(3-4)** (2015), 339-361.

On a class of holomorphic mappings in $\mathbb{C}^2$ related to bicomplex numbers

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In classic multidimensional complex analysis, a holomorphic mapping in $\mathbb{C}^2$ is just a pair of holomorphic functions of two complex variables with no relations between the functions themselves. It turns out that it is possible to introduce a Cauchy–Riemann–type relation in such a way that the arising subclass of holomorphic mappings possesses a rich theory quite similar to that of functions in one variable. It will be shown that a right way of treating it is via the so–called bicomplex analysis, that is, a study of derivable bicomplex functions. The main peculiarities of this approach will be presented.

Monogenic functions in finite-dimensional commutative associative algebras

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Let $\mathbb{A}_n$ be an arbitrary n-dimensional commutative associative algebra over the field of complex numbers. Let $e_1 = 1, e_2, \ldots, e_k$ with $2 \leq k \leq 2n$ be elements of $\mathbb{A}_n$ which are linearly independent over the field of real numbers. We consider monogenic (i. e. continuous and differentiable in the sense of Gateaux) functions of the variable $\sum_{j=1}^{k} x_j \, e_j$, where $x_1, x_2, \ldots, x_k$ are real, and obtain a constructive description of all mentioned monogenic functions by means of holomorphic functions of complex variables. It follows from this description that monogenic functions have Gateaux derivatives of all orders. The relations between monogenic functions and partial differential equations are investigated.

[1] V. S. Shpakivskyi, *Constructive description of monogenic functions in a finite-dimensional commutative associative algebra*, Adv. Pure Appl. Math., **7** (1) (2016), 63–75.

[2] V. S. Shpakivskyi, *Monogenic functions in finite-dimensional commutative associative algebras*, Zb. Pr. Inst. Mat. NAN Ukr., **12** (3) (2015), 251–268.

Fixed point theory and its applications

Organizer: E. Karapinar

A new extension of Darbo's fixed point theorem via α-admissible simulation functions

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In this talk, firstly, we introduce α_μ-admissible Z_μ-contraction and α_μ-admissibleN_μ-contraction via stimulation functions. Secondly, we prove some new fixed point theorems for defined class of contractions via α-admissible stimulation mappings. Our results extend some existing results.

The talk is based on collaborations with Hossein Monfared and Mahdi Azhini.

[1] A. Aghajani, R. Allahyari, M. Mursaleen, *A generalization of Darbo's theorem with application to the solvability of systems of integral equations*, J. Comput. Appl. Math. **260** (2014), 68-77.

[2] M. Asadi, E. Karapınar, and P. Salimi, *New Extension of p-Metric Spaces with Some fixed point Results on M-metric spaces*, Journal of Inequalities and Applications 2014, 2014:18.

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[4] F. Khojasteh, S. Shukla, S. Radenovic, *A new approach to the study of fixed point theory for simulation function*, Filomat, **29**(6) (2015), 1189-1194.

[5] A. Samadi, M.B. Ghaemi, *An extension of Darbo's theorem and its application*, Abstr. Appl. Anal., (2014) Article ID 852324.

Some Remarks On Kirk's Generalized Processes in Banach spaces

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In this talk, we establish a fixed point results for a finite sum of mappings defined on a convex set of an arbitrary Banach spaces and we study the convergence (resp. weak convergence) of generalized Kirk's processes associated to them.

The talk is based on collaborations with Hossein Monfared and Mahdi Azhini.

[1] K. Goebel and W. A. Kirk, *Topics in metric fixed point theory,* Cambridge Studies in Advanced Mathematics. First edition (1990).

[2] C. W. Groetsch, *A nonstationary iterative process for nonexpansive mappings,* Proc. Math. Soc., 43 (1) (1974), 155-158.

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[4] W. A. Kirk, *On successive approximations for nonexpansive mappings,* Glasgow. Math. J., Vol (2) (1), (1971), 6-9.

Fixed points of various contraction mappings on Branciari b-metric spaces

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In this paper, we discuss various types of α-admissible contractions mappings on Branciari b-metric spaces are introduced. We present theorems on the existence and uniqueness of fixed points of these mappings. We also conclude various consequences of these theorems and provide ilustrative examples.

The talk is based on collaborations with Selma Gülyaz Özyurt.

[1] S. Czerwik, *Contraction mappings in b-metric spaces*, Acta Math. Inform. Univ. Ostrav.,**1** (1993), 511.

[2] A. Branciari, *A fixed point theorem of Banach-Caccioppoli type on a class of generalized matric spaces*, Publ. Math. Debrecen, **57** (2000), 31-37.

[3] R. George, S. Radenovic, K.P. Reshma, S. Shukla, *Rectangular b-metric spaces and contraction principle*, J. Nonlinear Sci. Appl., **8** (2015), 10051013.

Study of Boundary value problems by a fixed point theorem

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In this talk, we investigate the existence of solutions for boundary value problems involving fractional derivatives. For this purpose, we transform the problem to an integral equation that we write as a sum of a contraction and a completely continuous operator then we use Krasnoselskii fixed point theorem to prove the existence of nontrivial solutions.

The talk is based on collaborations with Rabah Khaldi.

[1] T. A Burton, B. Zhang, Fixed points and fractional differential equations: examples, Fixed Point Theory, 10 No.2 (2009), 1-17.

[2] R. Khaldi, A. Guezane-Lakoud, Upper and lower solutions method for higher order boundary value problems, Progr. Fract. Differ. Appl. 3, No. 1, 53-57 (2017).

A short history of metric fixed point theory

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The emphasis of this talk will be on the historical origins of metric fixed point theory. We aim to list some basic results, and fundamental approaches to the theory. We also discuss the recent advances and improvements on the metric fixed point theory. The talk will be largely expository.

[1] Agarwal, R.P., Karapnar, E., O'Regan, D., Roldn-Lpez-de-Hierro, A.F., *Fixed Point Theory in Metric Type Spaces* Springer International Publishing (2015)

[2] S. Banach, *Sur les opérations dans les ensembles abstraits et leur applications aux équations intégrales* Fund. Math.,**3** 133-181, (1922).

[3] J. Caristi, *Fixed point theorems for mappings satisfying inwardness conditions, Trans. Amer. Math. Soc.,* **215** *241–251 (1976).*

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Generalization of Darbo's fixed point theorem via SR-functions with application to integral equations

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The aim of the current paper is introducing a generalization of Darbo's fixed point theorem based on $SR-$functions. In comparison with simulation function, $SR-$functions are able to cover the Meir-Keeler functions. Thus, the integral equations which are related to Meir-Keeler functions can be solved by our results. In the sequel, we find a solution for a functional integral equation

$$u(t) = \varphi(|u(t)|) + \int_0^t \frac{se^{-2t}|u(s)|}{u(s)+5} ds$$

where φ is a Meir-Keeler function, to support our results.

[1] F. Khojasteh, S. Shukla, S. Radenovic, *A new approach to the study of fixed point theorems via simulation functions*, Filomat, **29** (2015), 1189-1194.

[2] J. Chen X. Tang, *G*eneralizations of Darbo's fixed point theorem via simulation functions with application to functional integral equations, Journal of Computational and Applied Mathematics, **296** (2016), 564–575.

[3] A. Meir, E. Keeler, *A theorem on contraction mappings*, J. Math. Anal. Appl., **28** (1969), 326–329.

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Approximation of minimizers of convex functions defined on a complete geodesic spaces

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Finding a minimizer of given convex function is one of the simplest and most powerful problems in nonlinear analysis since it is related to other nonlinear problems such as equivalent problems, fixed point problems, variational inequality problems, saddle point problems, and others.
In this talk, we focus on approximation methods to a solution of this problem in the setting of complete geodesic spaces with a curvature bounded above. The notion of resolvent for convex function in this space plays an important role to generate an approximate sequence to its minimizer, and it is strongly related to fixed point thoery and its approximation theory. Our main result shows that the iterative sequence generated by the shrinking projection method with errors properly approximates to a minimizer of given function. We also show some recent developments related to this problem.

Fixed point theorems in ordered M-metric spaces

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Using control functions, we generalize some results of coupled fixed point theorems for nonlinear contractions to ordered M-metric spaces. Our conclusions extend and improve current results which is in partially ordered metric spaces by Lakshmikantam and Ćirić [6].

The talk is based on collaborations with Mehdi Asadi and Mahdi Azhini.

[1] R. P. Agarwal, M. A. El-Gebeily, D. O'Regan D. *Generalized contractions in partially ordered metric spaces*, Appl. anal., **87** (2008), 1-8.

[2] M. Asadi, E. Karapınar, and P. Salimi, *New Extension of p-Metric Spaces with Some fixed point Results on M-metric spaces*, Journal of Inequalities and Applications, **2014**, 2014:18.

[3] M. Asadi, *On Ekeland's Variational Principle in M-metric spaces*, Journal of Nonlinear and Convex Analysis, **17**(6) (2016), 1151-1158.

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[5] T. G., Bhaskar, V. Lakshmikantam, *Fixed point theorems in partially ordered metric spaces and applications*, Non linear Anal. TMA **65** (2006), 1379-1393.

[6] V. Lakshmikantham, L. Ćirić, *Coupled fixed point theorems for nonlinear contractions in partialy ordered metric spaces*, Non linear Anal., **70** (2009), 4341-4349.

[7] S.G. Mathews, *partial metric topology*, Proc. 8th Summer Conference on General topology and Applications Ann New York, Acad Sci., **728** (1994), 183-197.

[8] H. Monfared, M. Azhini, M. Asadi, *Fixed point results on M-metric spaces*, Journal of Mathematical Analysis, **7**(5) (2016), 85-101.

Fixed point theorems for mappings satisfying α-implicit contractive conditions in b-metric-like spaces

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In this study, by using α-admissible mappings and implicit contraction type conditions instead of the usual explicit contractive conditions, a general class of contractions have been defined and also common fixed point theorems have been obtained in the setting of b-metric like spaces. The obtained results unify, extend, generalize related common fixed point and fixed point theorems from the literature.

[1] M.A.Alghamdi, N. Hussain, P. Salimi, *Fixed point and coupled fixed point theorems on b-metric-like spaces*, J. Inequal. Appl., **2013**, (2013), 25 pages.

[2] V.Popa, *Fixed point theorems for implicit contractive mappings*, Stud. Cerc. St. Ser. Mat. Univ. Bacău, **7**, (1997), 127-133.

[3] B. Samet, C. Vetro, P. Vetro, *Fixed point theorems for $\alpha-\psi-$contractive type mappings*, Nonlinear Anal., **75**, (2012),

2154-2165.

[4] H. Aydi, A. Felhi, S. Sahmim, *On common fixed points for* (α, ψ)*-contractions and generalized cyclic contractions in b-metric-like spaces and consequences*, J. Nonlinear Sci. Appl., **9**, (2016), 2492-2510.

[5] E. Karapınar, B.Samet, *Generalized* $\alpha-\psi-$*contractive type mappings and related fixed point theorems with applications*, Abstr. Appl. Anal., **2012**, (2012), 17 pages.

Generalized implicit vector varitional inequality problems

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In this paper, we study existence result and properties of solution set of generalized implicit vector varitional inequality problems. For this purpose, at first two kinds of the upper sign continuity and pseudo monotonicity in the setting of multivalued bifunctions with moving cones are introduced. Moreover, with new definitions via KKM theory and without using Kakutani-Fan-Glickberg fixed point theorem, we stablish the properties of solution set of generalized implicit vector varitional inequality problems.

[1]Q.H. Ansari, A.P. Farajzadeh and S. Schaible, *Existence of solutions of strong vectror equilibrium problems*,Taiw. J. Math, 16 (1) (2012) 165-178.

[2] E. Blum, W. Oettli, *From optimization and varitional inequalities to equilibrium problems*, Journal of pure and applied abstracts Mathematics Student, 63 (93) 123-145.

[3] A.P. Farajzadeh, A. Amini-Harandi, *On generalized implicit vector variational inequality problems*, Indian J. Pure Appl. Math, 42 (2) (2011) 127-140.

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On Some Extensions of Nadler's Fixed Point Theorem

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In this talk, we give the notion of the pseudo-fixed point for multi-valued mappings which enable us to extend Nadler's theorem and other well known results in the literature.

[1] K. Iseki, *Multi-valued contraction mappings in complete metric spaces,* Rendiconti. del. Semin. Mat. della. Univ. Padova., 53 (1975), 15-19.

[2] J. R. Jachymski, *Caristi's fixed point theorem and selections of set-valued contractions,* J. Math. Anal. Appl. 227 (1998), 55-67.

[3] K. Kuratowski, *Topology, 1,* PWN, Warszawa, 1966.

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[6] S. Reich, *Fixed points of contractive functions,* Boll. Un. Mat. Ital., (4) 5 (1972), 26-42.

Concept of md-metric space and fixed point for the application in medical science

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In this paper we introduce a new type of metric space named as md-metric space. Once introduced, we then establish a Banach Contraction principle in this newly defined space (for area of curve) which deals with the study of Pharmacology and Pharmacodynamics (study of drugs and its effects on the body). The area under the plasma drug concentration-time curve reflects the actual body exposure to drug after administration of dose of the drug and is expressed in mg*h/l.

The talk is based on collaborations with Mahendra Singh Rathore.

Special interest group: IGPDE Harmonic analysis and partial differential equations

Organizers: M. Ruzhansky, S. Tikhonov, J. Wirth

On the solvability of the tracking problem with nonlinearly distributed control for the oscillation process

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In this talk we have investigated the tracking problem, where it is required to minimize the integral functional

$$J[u(t)] = \int_0^T \int_Q [V(t,x) - \xi(t,x)]^2 dxdt + 2\beta \int_0^T M[t,u(t)]dt, \quad \beta > 0,$$

on the set of solutions of the boundary value problem

$$V_{tt} - AV = \lambda \int_0^t K(t,\tau)V(\tau,x)d\tau + g[t,x,u(t,x)], \quad x \in Q \subset R^n, \quad 0 < t \leq T,$$

$$V(0,x) = \psi_1(x), \qquad V_t(0,x) = \psi_2(x), \qquad x \in Q,$$

$$\Gamma V(t,x) \equiv \sum_{i,j=1}^{n} a_{i,j}(x) V_{x_j} cos(\delta, x_i) + a(x)V(t,x) = 0, \quad x \in Q, \quad 0 < t \leq T.$$

Here $V(t,x) \in H(Q_T)$ is a function of state of the control process; $u(t,x) \in H(Q_T)$ is a function of the control; A is an elliptic operator; $H(Y)$ is a Hilbert space of functions defined on the set Y.
The questions of unique solvability of the tracking control problem with distributed optimal control for the oscillation process described by the Fredholm integral-dfferential equation in partial derivatives was investigated. The uniquely solution of this problem in the form of the triple $u^0(t)$, $V^0(t,x)$, $J[u^0(t)]$ was founded. Here $u^0(t)$ is the optimal control, $V^0(t,x)$ is the optimal process, $J[u^0(t)]$ is the minimum value of the functional.

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On the nonlinear Dirac equation with an electromagnetic potential

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In a joint work with M.Okamoto (Shinshu University, Nagano) we prove smoothing and Strichartz estimates for a Dirac equation perturbed by a large potential of critical decay and regularity. In the endpoint case, we prove suitable replacements of these estimates for data of additional angular regularity. As an application we deduce global well posedness and scattering for small data in the energy space with radial symmetry, or with additional angular regularity. Moreover, for a restricted class ofpotentials, we can extend our results to more general large data under the sole assumption of smallness of the Lochak-Majorana chiral invariants.

On $L_p - L_q$–boundedness of pseudodifferential operators with rough symbols on m–dimensional torus

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In the talk we will discuss $L_p - L_q$–boundedness for pseudo–differential operators on m–dimensional torus with smooth and rough symbols. In particular, we will formulate the Hörmander – Hounie type criterion of L_2–boundedness for pseudo–differential operators with symbols from periodic Hörmander classes and the Kenig – Staubach type theorem on L_p–boundedness of pseudo–differential operators with rough symbols with $1 \leq p \leq \infty$. Furthermore, we will discuss some sufficient conditions on $L_p - L_q$–boundedness for those operators with both type of symbols for $1 \leq p, q \leq \infty$.

Harmonic analysis methods in the theory of the kinetic transport equation

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We will see how various techniques from harmonic analysis can be applied to obtain Strichartz estimates and smoothing estimates for the velocity average of the solution of the kinetic transport equation. This will include applications of Fourier restriction estimates, cone multiplier estimates and certain multilinear geometric inequalities.

Approximation of autonomous equations by a fitted spectral method

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Solving highly oscillatory autonomous equations by conventional numerical integrators requires high computational effort if the frequency of oscillations is large. Alternative methods to treat this class of problems include techniques based on Magnus series expansions [1], coefficients perturbation [2], and exponential fitting (EF) ([3], [4]). In [3], I presented an exponentially fitted version of the spectral tau method (EFT) to solve a class of linear differential equations. The basic idea of EFT is to construct an approximate solution expressed as a linear combination of the exponentially weighted Legendre polynomials $\{e^{i\omega x}L_n(x);\ n = 0, 1, ..., N\}$, where $L_n(x)$ is the nth Legendre polynomial, and $\{\omega\}$ are frequencies whose the key role is to detect the sharp variations in y. In this talk, an EFT algorithm will be proposed for the integration of nonlinear autonomous systems of the form

$$y''_{xx} = F(y, y'_x)y'_x + G(y, y'_x)y, \quad x \geq x_0 \quad y(x_0) = \alpha_0, \quad y'_x(x_0) = \alpha'_0. \tag{10}$$

The adopted approach is to replace (10) by a sequence of linear equations, and then to apply EFT iteratively, until a certain prescribed tolerance is satisfied. This will generate a set of approximate solutions that converges, due to Newton-Kantorovich Theorem, to the exact solution $y(x)$ of (10). [1] I. Degani, J. Schiff, *RCMS: Right Correction Magnus Series approach for oscillatory ODEs.* J. Comput. Appl. Math. **193** (2006).

[2] V. Ledoux, M. Rizea, L. Gr. Ixaru, G. Vanden Berghe, M. Van Daele, *Solution of the Schrödinger equation by a high order perturbation method based on linear reference potential*, Comp. Phy. Com.**175**(2006), 424-439.

[3] M. K. El-Daou, *Exponentially weighted Legendre-Gauss Tau methods for linear second-order differential equations*, Comp. Math. with Appls **62** (2011), 51-64.

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The Cauchy problem for the generalized Zakharov-Kuznetsov equation in modulation spaces

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In this talk, we consider the well-posedness for the generalized Zakharov-Kuznetsov equation $\partial_t u + \partial_{x_1} \Delta u = \partial_{x_1}(u^{m+1})$ in modulation spaces $M^s_{p,q}(\mathbb{R}^n)$ with $n \geq 2$. In order to obtain the well-posedness, some linear estimates play crusial roles. In this talk, we mainly focus on the maximal function estimate and re-establish the estimate in the frame of modulation spaces. Then, as an application, we obtain the well-posedness in $M^0_{2,1}(\mathbb{R}^n)$ for $m \geq 4$, which is the result in a new class of functions which is not treated by that in the scaling critical Sobolev spaces.

A new conditions for the solvability of the nonlinear Fredholm integral equation

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In this paper we investigate questions of the unique solvability of the nonlinear Fredholm integral equation

$$\varphi(x) = \lambda \int_a^b K\big[x, t, \varphi(t)\big]\, dt. \tag{11}$$

It is established that if the following conditions are satisfied

(1) λ is not a characteristic number of the kernel $K_\psi\big[x, t, \psi_i(t, \lambda)\big]$;
(2) $\int_a^b K_\psi\big[x, t, \psi_i(t, \lambda)\big] \int_a^b K\big[t, s, f(s)\big]\, dsdt \equiv 0, \quad x \in [a, b],$
$i = 1, 2, ..., q.$

Then equation (1) is unique solvable. An algorithm of determining the unknown function $\psi_i(t, \lambda)$ was developed. Examples which sustain the theoretical conclusions were given.

[1] Akylbek Kerimbekov, *About the one method of solving nonlinear Fredgolm integral equations*, Abstract. V Congress of the Turkic World mathematicians, Kyrgyzstan, Issyk-Kul Aurora, 5-7 june, 2014 (2014), 121.

Singular integrals with product Cauchy kernels

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One of the purposes of our talk is to present necessary and sufficient conditions both for curves and weights ensuring the boundedness of maximal double Cauchy singular integral operator in weighted Lebesgue spaces. Almost everywhere existence of double Cauchy singular integrals will be discussed.

On approximation by multivariate Kantorovich–Kotelnikov sampling operators

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Approximation properties of multivariate Kantorovich-Kotelnikov sampling type operators generated by different band-limited functions are studied. In particular, a wide class of functions with discontinuous Fourier transform is considered. The L_p-rate of convergence for these operators is given in terms of the classical moduli of smoothness. Several examples of the Kantorovich-Kotelnikov operators generated by the sinc-function and its linear combinations are provided.

This is a joint work with Maria Skopina (St. Petersburg State University, Russia).

Optimal decay of Sobolev extremals on Stratified Lie groups and applications

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We obtain the regularity and the sharp asymptotic behavior at infinity of the extremal functions for the p-Sobolev inequality on Stratified Lie groups (see [4]). We recall that, in this abstract setting, the analytic expression of such functions is not known, except for the special case of the Heisenberg group H^n and $p = 2$. We shall discuss the semilinear Hardy-Sobolev case and the quasilinear case.
As firstly observed in [1], the exact rate of decay of Sobolev minimizers turns out to be the crucial ingredient in order to obtain existence results for critical growth problems, known as Brezis-Nirenberg type problems, characterized by lack of compactness. (see also [2], [3]).

[1] Annunziata Loiudice, *Semilinear subelliptic problems with critical growth on Carnot groups*, Manuscripta Math. **124** (2007), 247-259.

[2] Enrico Jannelli, Annunziata Loiudice, *Critical polyharmonic problems with singular nonlinearities*, Nonlinear Anal. **110** (2014), 77-96.

[3] Annunziata Loiudice, *Critical growth problems with singular nonlinearities on Carnot groups*, Nonlinear Anal. **126** (2015), 415-436.

[4] Annunziata Loiudice, *Optimal decay of Sobolev extremals on Carnot groups*, preprint 2017.

A Hardy-Sobolev inequality for the magnetic Laplacian

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We discuss a Hardy-Sobolev type inequality for the twisted Laplacian

$$\mathcal{L} = \sum_{j=1}^{n} \left[\left(i\partial_{x_j} + \frac{y_j}{2} \right)^2 + \left(i\partial_{y_j} - \frac{x_j}{2} \right)^2 \right]$$

on $\mathbb{C}^n$. The twisted Laplacian can be thought of as the magnetic Laplacian corresponding to a constant magnetic field, for a system of n non interacting particles in the plane. We obtain optimal Hardy-Sobolev type inequality of the form

$$\frac{1}{4}\int_{\mathbb{C}^n} |u|^2 w(z)\, dz \leq \int_{\mathbb{C}^n} |\nabla_{\mathcal{L}} u|^2\, dz$$

where the weight $w(z) = \left(\frac{|\nabla_{\mathcal{L}} E|^2}{E^2} + \frac{|z|^2}{4}\right)$ is given in terms of the fundametal solution E for the twisted Laplacian $\mathcal{L}$. The weight is *strong optimal*, in the sense that it can not be improved. The optimality relies on explicit construction of the fundamental solution for the twisted Laplacian in closed form, and it's asyptotic behaviour.

This talk is based on collaborative work with A. Adimurthi and Vijay Kumar Sohani.

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Composition in Weighted Modulation Spaces and Applications to Nonlinear PDEs

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In the theory of nonlinear partial differential equations we need to explain composition operators. We introduce a class of general weights for modulation spaces $\mathcal{M}^{w_*}_{p,q}(\mathbb{R}^n)$, which increase faster than any polynomial. For a particular scale of weights we establish analytic as well as non-analytic composition results in the spaces $\mathcal{M}^{w_*}_{p,q}(\mathbb{R}^n)$. Moreover, we study the existence of the product of two modulation spaces $M^s_{p,q}(\mathbb{R}^n)$ equipped with polynomial weights. This will give us the opportunity to treat the boundedness of composition operators acting on $M^s_{p,q}(\mathbb{R}^n)$. In both cases we discuss sufficient and necessary conditions. The obtained composition results bring us into position to treat several semi-linear Cauchy problems.

The talk is based on collaborations with Winfried Sickel (Friedrich Schiller University Jena, Germany).

Orthonormal Bases in the Orbits of Square-Integrable Representations of Nilpotent Lie Groups

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So far, wavelet and Gabor orthonormal bases of $L^2(\mathbb{R}^d)$ have been objects of applied harmonic analysis, which have proved useful for distinguishing and classifying certain features of functions. From an abstract point of view, they share a specific property: both lie in the orbit of a square-integrable unitary representation π of a locally compact group G. More specifically, they are of the form

$$\{\pi(\gamma)f \mid \gamma \in \Gamma\},$$

where Γ is a discrete subset (or even subgroup) of G.
The existence of such a basis for general locally compact G seems to be a new topic in abstract harmonic analysis and we can give a positive answer for a large class class of nilpotent Lie groups, which includes all graded groups with one-dimensional center. In the presence of a rational structure in the Lie algebra $\mathfrak{g}$, the set Γ can be chosen to be a uniform subgroup.

The talk is based on joint work with Karlheinz Gröchenig.

Pseudo-difference operators

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In this talk we discuss the calculus of pseudo-differential operators on the lattice $\mathbb{Z}^n$, which we can call *pseudo-difference operators*. An interesting feature of this calculus is that the phase space is compact ($\mathbb{T}^n$) so the question is *how to define symbol classes*? Nevertheless, we establish formulae for composition, adjoint, and parametrix for elliptic operators. We also give conditions for the ℓ^2, weighted ℓ^2, and ℓ^p boundedness. We describe a link to the toroidal quantization on the torus $\mathbb{T}^n$ ([5]), and apply it to give conditions for the membership in Schatten classes.
Furthermore, we will discuss a version of Fourier integral operators and give applications to solutions to difference equations on the lattice $\mathbb{Z}^n$.
Some properties of pseudo-differential operators on $\mathbb{Z}^n$ have been analysed e.g. in [1,2,3,4] but without symbolic calculus.

The talk is based on joint paper with Linda Botchway and Gael Kibiti from **AIMS Ghana** *(African Institute for Mathematical Sciences in Ghana)*

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[3] V. Rabinovich. Exponential estimates of solutions of pseudodifferential equations on the lattice $(h\mathbb{Z})^n$: applications to the lattice Schrödinger and Dirac operators. *J. Pseudo-Differ. Oper. Appl.*, 1(2):233–253, 2010.

[4] C. A. Rodriguez Torijano. L^p-estimates for pseudo-differential operators on $\mathbb{Z}^n$. *J. Pseudo-Differ. Oper. Appl.*, 2(3):367–375, 2011.

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Horisontal weighted Hardy-Rellich type inequalities on stratified Lie groups

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We present versions of horizontal weighted Hardy-Rellich type inequality and Caffarelli-Kohn-Nirenberg type inequality on stratified Lie groups and study some of its consequences. Our results reflect on many results previously known in special cases. Moreover, new Sobolev type spaces are defined on stratified Lie groups and proved an embedding theorems for these functional spaces.

[1] N. Badiale and G. Tarantello, *A Sobolev-Hardy inequality with applications to a nonlinear elliptic equation arising in astrophysics*, Arch. Ration. Mech. Anal., **163** (2002), 259–293.

[2] D. G. Costa., *Some new and short proofs for a class of Caffarelli-Kohn-Nirenberg type inequalities*, J. Math. Anal. Appl. **337** (2008), 311-317.

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[7] M. Ruzhansky and D. Suragan. *Layer potentials, Kac's problem, and refined Hardy inequality on homogeneous Carnot groups.* arXiv:1512.02547, (2015).

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[10] M. Ruzhansky and D. Suragan. *Local Hardy and Rellich inequalities for sums of squares of vector fields.* Adv. Diff. Equations, to appear, https://arxiv.org/abs/1601.06157.

On the solvability of the tracking problem under nonlinear boundary control of the thermal processes described by Volterra integro-differential equations

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In this talk we have investigated of the tracking problem, where it is required to minimize the quadratic functional

$$J[u(t)] = \int_0^T \int_0^1 [\nu(t,x) - \xi(t,x)]^2 dxdt + \beta \int_0^T u^2(t)dt, \quad \beta > 0, \tag{12}$$

on the set of solutions of the boundary value problem

$$\nu_t = \nu_{xx} + \lambda \int_0^t K(t,\tau)\nu(\tau,x)d\tau \quad 0 < x < 1, \ \ 0 < t \le T, \tag{13}$$

$$\nu(0,x) = \psi(x), \quad 0 < x < 1, \tag{14}$$

$$\nu_x(t,0) = 0, \quad \nu_x(t,1) + \alpha\nu(t,1) = p[t,u(t)], \quad 0 < t \le T. \tag{15}$$

Here $\nu(t,x) \in H(Q)$ is a function of state of the control process; $u(t) \in H(0,T)$ is a function of the control; $H(Y)$ is a Hilbert space of functions defined on the set Y.
An algorithm for constructing solutions tracking problem of the nonlinear optimization for the thermal process described by the Volterra integro-differential equation in partial derivatives was developed.

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Hardy type integral inequalities involving many functions for $0<p<1$

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It is well known the classical Hardy's inequality:

$$\int_0^\infty \Big(\frac{F(x)}{x}\Big)^p dx < \Big(\frac{p}{p-1}\Big)^p \int_0^\infty \Big(f(x)\Big)^p dx. \tag{16}$$

Where $p>1$, f non negative measurable function. Our aim is to generalize the corresonding weighted inequality of (1) with $0<p<1$ for many functions.

Theorem 1 *Let $f_1, f_2, ..., f_n$ n functions measurable and non negative, $0<p<1$.*
If $-\frac{1}{p}<\alpha<1-\frac{1}{p}$ and for every $i=1,2,...n$ f_i is non increasing, then

$$\int_0^\infty \Big(x^{n\alpha}(Hf_1)(x)(Hf_2)(x)...(Hf_n)(x)\Big)^{\frac{p}{n}} dx \leq$$
$$\frac{1}{n^p}\Big(1-\frac{1}{p}-\alpha\Big)^{-1}\int_0^\infty x^{\alpha p}\Big(f_1(x)+f_2(x)+...+f_n(x)\Big)^p dx. \tag{17}$$

If $\alpha<-\frac{1}{p}$ and for every $i=1...n$ f_i is non decreasing, then

$$\int_0^\infty \Big(x^{n\alpha}(Hf_1)(x)(Hf_2)(x)...(Hf_n)(x)\Big)^{\frac{p}{n}} dx \leq$$
$$\frac{p}{n^p}B(p,-\alpha p)\int_0^\infty x^{\alpha p}\Big(f_1(x)+f_2(x)+...+f_n(x)\Big)^p dx. \tag{18}$$

If $\alpha>1-\frac{1}{p}$ and for every $i=1...n$ f_i is non increasing, then

$$\int_0^\infty (x^{n\alpha}\Big(H^*f_1)(x)(H^*f_2)(x)...(H^*f_n)(x)\Big)^{\frac{p}{n}} dx \leq$$
$$\frac{p}{n^p}B(p,\alpha p+1-p)\int_0^\infty x^{\alpha p}\Big(f_1(x)+f_2(x)+...+f_n(x)\Big)^p dx. \tag{19}$$

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Nikol'skij-Besov Spaces Built on Morrey Spaces

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The first half of the talk will be devoted to the introduction of periodic Nikol'skij-Besov spaces $\mathcal{N}^s_{u,p,q}(\mathbb{T}^d)$ built on Morrey spaces. We shall discuss characterizations by approximation as well as characterizations by differences. A few remarks to interpolation of theses classes will be made as well. In a second part we shall deal with embeddings of these classes. In particular, we will be interested in the asymptotic behaviour of the approximation numbers of embeddings of theses spaces into $L_\infty(\mathbb{T}^d)$.
Key words: Besov spaces; Morrey spaces, Morrey smoothness spaces, embeddings, approximation numbers.

The talk is based on collaborations with M. Sautbekova (Astana) and Z. Baituyakova (Astana).

[1] M. Sautbekova and W. Sickel, *Strong summability of Fourier series and Morrey spaces*, Analysis Mathematica **40** (2014), 31-62.

[2] Z. Baituyakova and W. Sickel, *Strong summability of Fourier series and generalized Morrey spaces*, Analysis Mathematica (to appear).

Spectral Comparison of Smoothing Estimates

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In this talk, smoothing estimates for abstract Schrödinger equations

$$\begin{cases} i\partial_t u(t) = Hu(t) \\ u(0) = f \in \mathcal{H} \end{cases}$$

will be discussed, where H is a self-adjoint operator on a Hilbert space $\mathcal{H}$, by developping *comparison principle* which was first introduced in [1] for the case

$$\mathcal{H} = L^2(\mathbb{R}^n), \qquad H = a(D_x)$$

with real-valued functions $a(\xi)$. Comparison principle is a general rule to derive a smoothing estimate for $u(t) = e^{itH}$ from that for $v(t) = e^{it\widetilde{H}}$. We will allow to compare them in terms of their spectral densities. A number of applications will be given, in particular for Schrödinger equations on the Euclidean space with potentials.

The talk is based on collaborations with Matania Ben-Artzi (Hebrew University) and Michael Ruzhansky (Imperial College London).

[1] M. Ruzhansky., M. Sugimoto, *Smoothing properties of evolution equations via canonical transforms and comparison principle*, Proc. Lond. Math. Soc. **105** (2012), 393–423.

Elements of potential theory on Carnot groups

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We talk about elements of potential theory for the sub-Laplacian on homogeneous Carnot groups. In particular, we show continuity of the single layer potential and establish the Plemelj type jump relations for the double layer potential. We present an adapted divergence formula as well as the sub-Laplacian versions of Green's first and second formulae on homogeneous Carnot groups. As a consequence, we derive the formula for the trace to smooth surfaces of the Newton potential for the sub-Laplacian. We also give a refined local version of Hardy's inequality.

The talk is based on collaborations with Michael Ruzhansky [1].

[1] M. Ruzhansky, D. Suragan, *Layer potentials, Kac's problem, and refined Hardy inequality on homogeneous Carnot groups*, Advances in Mathematics **308** (2017), 483-528.

Nonharmonic Analysis and its Applications

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Here, we will talk about Nonharmonic Analysis and its applications into PDEs and nonlinear equations. In particular, we derive an explicit formula for the quantization of pseudo–differential operators generated by model operators.

The talk is based on collaborations with Michael Ruzhansky.

On stability of inverse scattering problem on a sun-type graph

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We investigate an inverse scattering problem for a Sturm-Liouvile operator on a sun-type graph which consists of a loop and half lines joined at the different points of the loop with natural Kirchhoff matching conditions at vertices. We have deduced Marchenko equation ([1]-[3]) which allowed us to prove the uniqueness theorems, provide a reconstruction procedure for the potential on the half lines and investigate the conditional stability of the inverse scattering problem, based on the knowledge of scattering matrix and point spectrum.

The talk is based on collaborations with Prof. K.Mochizuki, Tokyo Metropolitan University (Emiritus).

[1] V. Marchenko, K. Mochizuki and I. Trooshin, *Inverse scattering on a graph containing circle* (Analytic methods of analysis and differential equations: AMADE 2006)(Cambridge: Camb. Sci. Publ), (2008), 237-243.

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Quantizations and time-frequency transforms

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A Cohen class time-frequency transform of signals $u, v : \mathbb{R} \to \mathbb{C}$ is a time-frequency invariant sesquilinear mapping $(u, v) \mapsto C(u, v)$, where the time-frequency distribution

$$C[u] = C(u, u) : \mathbb{R} \times \widehat{\mathbb{R}} \to \mathbb{R}$$

can be thought as a phase-space energy density of u. For instance, all the spectrograms are such energy densities. We study properties of different time-frequency transforms C and their related pseudo-differential operator quantizations $a \mapsto a_C$ defined by the Hilbert space duality

$$\langle u, a_C v\rangle_{L^2(\mathbb{R})} = \langle C(u, v), a\rangle_{L^2(\mathbb{R}\times\widehat{\mathbb{R}})}.$$

We also present computed examples from acoustic signal processing, quantum mechanics and medical sciences. When and how often something happens in signals? By properly quantizing these questions, we obtain the Born–Jordan transform.

[1] V. Turunen, *Born–Jordan time-frequency analysis.* Harmonic Analysis and Nonlinear Partial Differential Equations, eds. H. Kubo and M. Sugimoto. *RIMS Kôkyûroku Bessatsu* **B56**, Research Institute of Mathematical Sciences, Kyoto University (2016), 107–186.

On Dissipative Nonlinear Evolutional Pseudo-Differential Equations

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In this paper we consider the Cauchy problem for the dissipative evolutionary pseudo-differential equation

$$\partial_t u + A(x, D)u = F\big((\partial_x^\alpha u)_{|\alpha|\le\kappa}\big), \quad u(0, x) = u_0(x),$$

where $A(x, D)$ is a dissipative pseudo-differential operator and $F(z)$ is a multi-polynomial. We will develop the uniform decomposition techniques to both physical and frequency spaces to study its local well posedness in modulation spaces $M^s_{p,q}$ and in Sobolev spaces H^s. Moreover, the local solution can be extended to a global one in L^2 and in H^s $(s > \kappa+d/2)$ for certain nonlinearities.

The talk is based on collaborations with Mingjuan Chen and M. W. Wong.

Non-commutative Fourier multipliers, differential operators and calculus

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Classical Fourier analysis on Euclidean space is the main tool to study translation invariant operators. Replacing the translation group by more general Lie groups, i.e. considering different notions of symmetries, leads us to questions concerning non-commutative Fourier multipliers and their basic properties. These operators are characterised by their action on irreducible representations of the group and lead to natural notions of symbols. One of the key questions arising for such operators are boundedness properties and their characterisation in terms of the associated symbols.
We review some of the recent results in this direction and show how estimates for and the calculus of operators on groups is intrinsically connected to the representation theory of the group.

New classes of the index transforms and higher order PDEs

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We discuss new index transforms, which are associated with the modified Bessel functions as the kernel. The boundedness and invertibility are examined for these operators in the Lebesgue weighted spaces. Inversion theorems are proved. Important particular cases are exhibited. The results are applied to solve boundary value problems for higher order PDE, involving the Laplacian.

Hardy inequalities for Baouendi-Grushin operator with Aharonov-Bohm type magnetic field and for twisted Laplacian

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In [1] for the quadratic form of the following magnetic Grushin operator

$$G_{\mathcal{A}} = -(\nabla_G + i\beta\mathcal{A}_0)^2,$$

the following Hardy inequality for $-1/2 \leq \beta \leq 1/2$ was proved:

$$\int_{\mathbb{R}^3} |(\nabla_G + i\beta\mathcal{A}_0)f|^2 dzdt \geq (1+\beta^2)\int_{\mathbb{R}^3} \frac{|z|^2}{d^4}|f|^2 dzdt,$$

where $\mathcal{A}_0 = (\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4) = \left(-\frac{\partial_y d}{d}, \frac{\partial_x d}{d}, -2y\frac{\partial_t d}{d}, 2x\frac{\partial_t d}{d}\right)$, $\nabla_G = (\partial_x,\, \partial_y,\, 2x\partial_t,\, 2y\partial_t)$ with $z = (x, y)$, $|z| = \sqrt{x^2+y^2}$, $\beta \in \mathbb{R}$ is a flux and $d(z,t) = (|z|^4 + t^2)^{\frac{1}{4}}$ is the Kaplan distance.
In this talk, we discuss the more general form of this result with sharp constant. For this, we give refinements of the known Hardy inequalities for the Baouendi-Grushin operator involving radial derivatives in some of the variables. We are also interested in the Hardy inequalities for the quadratic form of the Laplacian with the Landau Hamiltonian magnetic field. Moreover, the estimates for the remainder terms of these inequalities are obtained.

The talk is based on collaborations with Professors A.Laptev and M.Ruzhansky [2].

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Nonlinear PDE

Organizers: V. Georgiev, T. Ozawa

Spectral stability of small amplitude solitary waves of the Dirac equation with the Soler-type nonlinearity

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We study the point spectrum of the linearization at a solitary wave solution $\phi_\omega(x)e^{-i\omega t}$ to the nonlinear Dirac equation in $\mathbb{R}^n$, $n \geq 1$, with the nonlinear term given by $f(\psi^*\beta\psi)\beta\psi$ (known as the Soler model). We focus on the spectral stability, that is, the absence of eigenvalues with nonzero real part, in the non-relativistic limit $\omega \lesssim m$, in the case when $f \in C^1(\mathbb{R} \setminus \{0\})$, $f(\tau) = |\tau|^k + O(|\tau|^K)$ for $\tau \to 0$, with $0 < k < K$. For $n = 1$ and $n \geq 3$, we prove the spectral stability of small amplitude solitary waves ($\omega \lesssim m$) for the charge-subcritical cases $k \lesssim 2/n$ ($1 < k \leq 2$ when $n = 1$) and for the "charge-critical case" $k = 2/n$, $K > 4/n$.
An important part of the stability analysis is the proof of the absence of bifurcations of nonzero-real-part eigenvalues from the embedded threshold points at $\pm 2m$i. Our approach is based on constructing a new family of exact bi-frequency solitary wave solutions in the Soler model, on using this family to determine the multiplicity of $\pm 2\omega$i eigenvalues of the linearized operator, and on the analysis of the behaviour of "nonlinear eigenvalues" (characteristic roots of holomorphic operator-valued functions).

The talk is based on collaborations with Andrew Comech (Texas A&M University (USA), St. Petersburg State University (Russia) & Institute for Information Transmission Problems (Russia)): Boussaid, N., & Comech, A. 2017, arXiv:1705.05481

N. Boussaid and A. Comech, *Soler model: stability, bi-frequency solitons, and* SU(1,1), talk at Emergent Paradigms in Nonlinear Complexity workshop (Santa Fe, NM) (2015).
N. Boussaïd and A. Comech, *On spectral stability of the nonlinear Dirac equation*, J. Funct. Anal. **271** (2016), pp. 1462–1524.
N. Boussaid and A. Comech, *Nonrelativistic asymptotics of solitary waves in the Dirac equation with the Soler-type nonlinearity*, SIAM Journal on Mathematical Analysis (2017), to appear, arXiv:1606.07308.

A class of the elliptic systems with even functionals

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We get a result that shows the existence of infinitely many solutions for a class of the elliptic systems involving subcritical Sobolev exponents nonlinear terms with even functionals on the bounded domain with smooth boundary. We get this result by variational method and critical point theory induced from invariant subspaces and invariant functional.

The talk is based on collaborations with Tacksun Jung.

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Incompressible limits for generalisations to symmetrisable systems

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We start from systems of fluid dynamics such as the Euler system of gas dynamics, or from semiconductor systems with and without quantum effects. The presence of quantum effects will introduce a mixed-order structure (in the sense of Douglis–Nirenberg) to the stationary system. It is well-known that there are various mechanisms which can make the flow incompressible, such as the low Mach number limit, or the quasi-neutral limit in case of systems of transport of electrically charged particles.
The talk presents a general functional analytical framework for handling all these limits, in the context of symmetrisable systems. The mixed-order structure is being taken care of by choosing an operator-valued symmetrising matrix.

Three Nontrivial Solutions for a Nonlinear Fractional Laplacian Problem

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We consider the problem

$$\begin{cases} (-\Delta)^s u = f(x,u) & \text{in } \Omega \\ u = 0 & \text{in } \Omega^c, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^N$ $(N > 1)$ is a bounded domain with a C^2 boundary, $s \in (0,1)$, and $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function. The fractional Laplacian operator is defined for any sufficiently smooth function $u : \mathbb{R}^N \to \mathbb{R}$ and all $x \in \mathbb{R}$ by

$$(-\Delta)^s u(x) = C_{N,s} \lim_{\varepsilon \to 0^+} \int_{\mathbb{R}^N \setminus B_\varepsilon(x)} \frac{u(x) - u(y)}{|x-y|^{N+2s}} \, dy,$$

where $B_\varepsilon(x)$ is the open ball of radius $\varepsilon > 0$ centered at x and $C_{N,s} > 0$ is a suitable normalization constant. For the existence of three nontrivial solutions of problem (1.1), we make use of the second deformation theorem and some spectral properties of $(-\Delta)^s$ if $f(x,.)$ is sublinear at infinity and make use of the mountain pass theorem and Morse theory if $f(x,.)$ is superlinear at infinity.

The talk is based on collaborations with Antonio Iannizzotto.

[1] Fatma Gamze Duzgun, Antonio Iannizzotto, *Three nontrivial solutions for nonlinear fractional Laplacian equations*, Advances in Nonlinear Analysis, ISSN (Online) 2191-950X, ISSN (Print) 2191-9496, DOI: https://doi.org/10.1515/anona-2016-0090 (2016).

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Scattering for Nonlinear Klein-Gordon equations on product spaces. Part II: large data scattering.

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In this talk, we will study the large time behavior of nonlinear dispersive PDEs, and will focus on the particular case of the Klein-Gordon equation. The behavior of such equations is well-known on the euclidean space $\mathbb{R}^d$: one can make a comparison, in some sense, with solutions of the linear equation ("scattering"). On the other hand, this kind of phenomenon does not appear on compact riemannian manifolds such as the $k-$dimensional flat torus $\mathbb{T}^k$. Our aim is to understand the leading behavior when one mixes the spaces, namely, if the equation is posed on a product space of the form $\mathbb{R}^d \times \mathbb{T}^k$.

After having proved, in Part I, the validity of the Strichartz estimates tool, in this second part of the talk, the case of large data scattering will be handle. The proof of the scattering property for defocusing power nonlinearity will be given by employing a concentration-compactness and rigidity argument.

The talk is based on collaborations with Professor N. Visciglia (University of Pisa) and Dr. L. Hari (UFC, Besançon).

Higher order fractional Leibniz rule

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One of the most important tools to obtain local well-posedness of nonlinear equations of mathematical physics is based on the bilinear estimate of the form

$$\|D^s(fg)\|_{L^p} \leq C\|D^s f\|_{L^{p_1}}\|g\|_{L^{p_2}} + C\|f\|_{L^{p_3}}\|D^s g\|_{L^{p_4}}, \tag{20}$$

where $D^s = (-\Delta)^{s/2}$ is a Fourier multiplier with symbol $|\xi|^s$ and $s \in \mathbb{R}$, $L^p = L^p(\mathbb{R}^n)$ and $f, g \in \mathcal{S}(\mathbb{R}^n)$. A typical domain for parameters $s, p, p_j, j = 1, \cdots, 4$, where (20) is valid is

$$s > 0, \quad 1 < p, p_1, p_2, p_3, p_4 < \infty, \quad 1/p = 1/p_1 + 1/p_2 = 1/p_3 + 1/p_4.$$

Classical proof can be found in [1]. Another estimate showing the flexibility in the redistribution of fractional derivatives can be deduced. More precisely, Kenig, Ponce, and Vega [2] obtained the estimate

$$\|D^s(fg) - fD^s g - gD^s f\|_{L^p} \leq C\|D^{s_1} f\|_{L^{p_1}}\|D^{s_2} g\|_{L^{p_2}}, \tag{21}$$

provided

$$0 < s = s_1 + s_2 < 1, \quad s_1, s_2 \geq 0, \quad 1 < p, p_1, p_2 < \infty, \quad 1/p = 1/p_1 + 1/p_2.$$

One can interpret the bilinear form

$$\mathrm{Cor}_s(f, g) = fD^s g + gD^s f$$

as a correction term such that for any redistribution of the order s of the derivatives. In this talk, we show $\mathrm{Cor}_s(f, g)$ for $s \geq 1$.

The talk is based on collaborations with professors Vladimir Georgiev and Tohru Ozawa.

[1] L. Grafakos, Z. Si, *The Hörmander multiplier theorem for multilinear operators*, J. Reine Angew. Math., **668**(2012), 133–147.

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Scattering for Nonlinear Klein-Gordon equations on product spaces. Part I: small data scattering

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In this talk, we will study the large time behavior of nonlinear dispersive PDEs, and will focus on the particular case of the Klein-Gordon equation. The behavior of such equations is well-known on the euclidean space $\mathbb{R}^d$: one can make a comparison, in some sense, with solutions of the linear equation ("scattering"). On the other hand, this kind of phenomenon does not appear on compact riemannian manifolds such as the $k-$dimensional flat torus $\mathbb{T}^k$. Our aim is to understand the leading behavior when one mixes the spaces, namely, if the equation is posed on a product space of the form $\mathbb{R}^d \times \mathbb{T}^k$.

In this first part of the talk, we will prove that Strichartz estimates are available even on a product space with trapped variables and will deduce small data scattering. The case of large data scattering will be handle in Part II and it will rely on a concentration-compactness and rigidity scheme.

The talk is based on collaborations with Professor N. Visciglia (University of Pisa) and L. Forcella (Scuola Normale Superiore, Pisa).

Elliptic system with singular nonlinearity

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We investigate multiplicity of solutions for the elliptic system with singular nonlinearity. We obtain a theorem which shows that the elliptic system with some singular nonlinearity has infinitely many solutions. We get this result by using the variational method, critical point theory and homology theory.

The talk is based on collaborations with Q-Heung Choi.

[1] R. Bott, *Nondegenerate critical manifolds,* Ann. of Math. **60**, 248-261(1954).

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Well-posedness for the Cauchy problem of the Klein-Gordon-Zakharov system in 2D and 3D

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We consider the Cauchy problem of 2D and 3D Klein-Gordon-Zakharov system (KGZ) with very low regularity initial data.

$$\begin{cases}(\partial_t^2 - \Delta + 1)u = -nu, \\ (\partial_t^2 - c^2\Delta)n = \Delta|u|^2.\end{cases}$$

(KGZ) describes the interaction of the Langmuir wave and the ion acoustic wave in a plasma. We prove the local in time well-posedness for (KGZ) by using the Fourier restriction norm method. We show the bilinear $X^{s,b}$ estimates which are crucial to get the well-posedness. We employ the nonlinear version of the classical Loomis-Whitney inequality which enable us to estimate bad bilinear interactions.

Local energy decay estimates for wave equation on exterior domains

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This talk is devoted to studying the local energy decay estimates for 3-dimensional wave equation to the initial-boundary value problem on exterior domains. The geometrical assumptions on domains are rather general, for example non-trapping condition is not imposed in the local energy decay result.

The talk is based on collaborations with Vladimir Georgiev of Pisa.

Smoothing properties and scattering for magnetic Klein-Gordon equations in exterior domain with time dependent perturbations

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This talk deals with the smoothing properties and scattering of solutions to magnetic Klein-Gordon equations in exterior domain with time dependent small perturbations. Smoothing properties based on the resolvent estimates will reinforce the abstract scattering theory, and our concrete problems are treated in this framework.

Mixing and limit theorems for a damped nonlinear wave equation with space-time localised noise

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In this talk we deal with a damped nonlinear wave equation perturbed by a space-time localised noise, in a bounded domain with a smooth boundary. It is assumed that the random perturbation is non degenerate, it has a support localized in space and time, and its law is periodic in time. Under these hypotheses, we show that the Markov process generated by the solution possesses a unique stationary measure, and the law of any solution converges to this measure with exponential rate in the dual-Lipschitz norm. The proof is based on a coupling approach and a property of stabilisation to a non stationary solution of a nonlinear wave equation. Moreover, we use these results to prove that the solution of the equation fulfills the strong law of large numbers along with the central limit theorem and to estimate the corresponding rates of convergence. Finally, under a non-degeneracy assumption, we prove a local large deviation principal for this system.

The talk is based on collaborations with PDEs Laboratory, Department of Mathematics, Faculty of sciences El Manar Tunis.

[1] K. Ammari and T. Duyckaerts and A. Shirikyan, *Local feedback stabilisation to a non-stationary solution for a damped non-linear wave equation*, Math. Control Related. Fields, 6 (2016), pp. 1–25.

[2] V. Barbu and G. Da Prato, *The stochastic nonlinear damped wave equation*, Appl. Math. Optim., 46 (2002), pp. 125–141.

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[4] S. Kuksin and A. Shirikyan, *Mathematics of two-Dimensional Turbulence*, Cambridge University Press, Cambridge, 2012.

Ill-posedness for the Cauchy problem of the compressible Navier-Stokes system in the critical Besov space – the barotropic case

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We consider the Cauchy problem of the compressible Navier-Stokes system of barotropic type:

$$\begin{cases} \partial_t \rho + \text{div } ((1+\rho)u) = 0, & t>0, x\in\mathbb{R}^n, \\ \partial_t u + (u\cdot\nabla)u + \dfrac{1}{1+\rho}\nabla P(\rho) & \\ \qquad = \dfrac{1}{1+\rho}\big(\mu\Delta u + (\mu+\lambda)\nabla \text{div } u\big) \equiv \dfrac{1}{1+\rho}\mathcal{L}u, & t>0, x\in\mathbb{R}^n, \\ P(\rho) = \rho^\alpha & t>0, x\in\mathbb{R}^n, \\ \rho(0,x)=\rho_0(x), \quad u(0,x)=u_0(x), & t=0, x\in\mathbb{R}^n, \end{cases} \tag{cNS}$$

where ρ, u are the unknown functions denoting the density and velocity of fluids, λ, μ are the Lamé constants and $\alpha \geq 1$. Applying the Fujita-Kato principle, it is known that the system is uniquely solvable in the scaling critical Besov spaces $\dot{B}^{\frac{n}{p}}_{p,1}(\mathbb{R}^n)\times\dot{B}^{-1+\frac{n}{p}}_{p,1}(\mathbb{R}^n)$ for $p<2n$ (Danchin [Inv. Math. 2000], Haspot [JDE 2015]), and it is ill-posed for $p>2n$ (Chen-Miao-Zhang [RevMIAmeri 2015]). We show the strong discontinuity for the solution corresponding integral equation to (cNS) occurs and it is ill-posed in the critical homogeneous Besov space $\dot{B}^{\frac{n}{p}}_{p,1}(\mathbb{R}^n)\times\dot{B}^{-1+\frac{n}{p}}_{p,1}(\mathbb{R}^n)$ with $p=2n$. The argument depends on finding a typical initial data that develops the diverging second order iteration for the related integral equation. The striking difference from the other case of the exponent $p>2n$ is that the worst term from the convection term is canceling with the term from the linearized quasi-linear viscosity term and we need finer analysis to show the strong discontinuity for the solution.

This is a joint work with Tsukasa Iwabuchi.

Partial differential equations of quantum physics

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The goal of the talk is to give an informal overview of the active area of partial differential equations of quantum physics. I will indicate very briefly some of the recent results. I will begin with the key many-body Schödinger equation and then proceed to the effective nonlinear equations: the Hartree, Hartree-Fock, Hartree-Fock-Bogolubov and Bogolubov-de Gennes equations. The latter two describe quantum fluids: superfluids and superconductors. If time permits I will say a few words about the Ginzburg-Landau and Yang-Mills-Higgs equations and their relations to geometry.

The talk is based on joint work with Li Chen and with V. Bach, S. Breteaux, Th. Chen and J. Fröhlich. We might mention results with T. Tsaneteas and with D. Chouchkov, N. Ercolani and S. Rayan.

Blow-up for semilinear damped wave equations with super-Fujita exponent

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The blow-up result for semilinear damped wave equations is well-studied for sub-Fujita exponent. On the other hand, there has been only one for super-Fujita exponent by Wakasugi [3] in the non-effective case of the scale invariant damping. In this talk, I would like to introduce you that the blow-up result in [1] can be obtained not only for super-Fujita exponent, but also for bigger one which is closely related to Strauss exponent, the critical number for non-damped case. Such a result is established for a new domain of the constant in the scale invariant damping term, which includes a part of the outside of non-effective range. Moreover, I would like to mention to sub-Strauss blow-up result in [2] for semilinear damped wave equations in the scattering range.

The talk is based on collaborations with Lai Ning-An (Lishui Univ. China) and Kyouhei Wakasa (Muroran Institute of Technology, Japan).

[1] Lai N.-A., Takamura H., Wakasa K., *Blow-up for semilinear wave equations with the scale invariant damping and super-Fujita exponent*, arXiv1701.03232.

[2] Lai N.-A., Takamura H., *Blow-up for semilinear damped wave equations in the scattering range with sub-Strauss exponent*, in preperation.

[3] Wakasugi Y., *Critical exponent for the semilinear wave equation with scale invariant damping*, Fourier analysis, 375-390, Trends Math., Birkhäuser/Springer, Cham, (2014).

Numerical modeling of the one Musket – Leverett's filtration model by Monte Carlo methods

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We consider the filtration problems with non-Newtonian properties and generalization of Darcy laws for incompressible components of mixture for the nonlinear law of medium resistance. This results in consideration of two nonlinear form of Darcy laws:

$$\mathbf{v}_i = \mathbf{u}_i + (-1)^i \mathbf{F}_0, \;\; \mathbf{F}_0 = \Phi(s, w)\mathbf{w}; \;\; \mathbf{v}_i = \Phi(w)\mathbf{u}_i, \;\; i = 1, 2, \tag{22}$$

that corresponds to system of equations (in linear case) - Musket-Leverett's filtration model - relative to saturation $s(t,x)$ and pressure $p(t,x)$:

$$m\partial_t s(t,x) = \operatorname{div} \Phi_n\big(K_0 \operatorname{grad} s(t,x) - b\mathbf{v} + \mathbf{F} + (1-n)\mathbf{F}_0\big), \;\; n = 0, 1, \tag{23}$$

$$\operatorname{div} \Phi_n\big(K \operatorname{grad} p(t,x) + \mathbf{f}\big) = 0, \;\; n = 0, 1. \tag{24}$$

Here $\Phi_i = p_i + \rho_i g h$ is a potential, $\mathbf{g} = g\nabla h$, $\mathbf{u}_i = -K_i\big(\nabla p_i + \rho_i \mathbf{g}\big)$, $w = |\mathbf{w}|$, the $\mathbf{w}$ is equal to one of the $\mathbf{u}_i$, $\mathbf{v}$ or with $K_0 \nabla p_c$, what corresponds to an assumption of realization of typical nonlinear Darcy law for component $\mathbf{u}_i$, also to an assumption of proportionality of interaction forces to the total consumption of the mixture or to an assumption of prioritized effect of capillary forces on process of component interaction. Initial boundary problem for (2), (3). The filtration stream is given in a finite domain Ω with piecewise-smooth boundary $\partial\Omega$. This boundary $\partial\Omega$ may be divided to several connected components $\partial\Omega^i$. Let $Q = [0,T] \times \Omega$, $G^i = [0,T] \times \partial\Omega^i$, $\mathbf{n}$ is external normal to $\partial\Omega$. The non-fluxion condition on $\partial\Omega^0$ for two phases is equivalents to:

$$\mathbf{v} \cdot \mathbf{n} = \mathbf{v}_1 \cdot \mathbf{n} = 0, \;\; (t,x) \in G^0 = [0,T] \times \partial\Omega^0. \tag{25}$$

Then

$$p = p_0(t,x), \;\; s = s_0(t,x), \;\; (t,x) \in G^2 = [0,T] \times \partial\Omega^2, \tag{26}$$

$$-\big(K\nabla p + \mathbf{f}\big) \cdot \mathbf{n} \equiv \mathbf{v} \cdot \mathbf{n} = R(t,x), \;\; (t,x) \in G^1 = [0,T] \times \partial\Omega^1, \tag{27}$$

$$-\big(K_0 a \nabla s + K_1 \nabla p + \mathbf{f}_0\big) \cdot \mathbf{n} \equiv \mathbf{v}_1 \cdot \mathbf{n} = bR(t,x), \;\; (t,x) \in G^1. \tag{28}$$

It is sufficient to set the initial condition only for saturation $s(t,x)$:

$$s(0,x) = s_0(x), \;\; x \in \Omega. \tag{29}$$

Here

$$a = -\partial_s p_c \frac{k_{01}k_{02}}{k}, \; k = k_{01} + k_{02}, \; b = \frac{k_{01}}{k} \equiv K_1 K^{-1}, \; \mathbf{f}_0 = K_1 \int_s^1 \nabla \partial_s p_c \frac{k_{02}}{k},$$

$$K_i = k_{0i} K_0, \; i = 1, 2, \; \mathbf{f} = K K_1^{-1} \mathbf{f}_0 + K_2 \nabla p_c + K_2(\rho_2 - \rho_1)\mathbf{g}, \; K = K_1 + K_2,$$

$\mathbf{F} = \mathbf{f}_0 - b\mathbf{f} = -k_{01}k_{02}k^{-1}K_0\big(\nabla p_c + (\rho_2 - \rho_1)\mathbf{g}\big)$. [1]. The equations (2), (3) and initial boundary conditions (4) – (8) are discretized only by variable t by means of implicit scheme. As a result we have the following equations for time layers t_{n+1} with respect to $s^{n+1}(x)$ and $p^{n+1}(x)$:

$$L_s s^{n+1} = D^n(x), \;\; x \in \Omega, \tag{30}$$

$$L_p p^{n+1} = C^n(x), \;\; x \in \Omega. \tag{31}$$

If input data (coefficients and functions) of problems (2), (3), (4) – (8) provides ellipticity of operators L_s and L_p, then we have the following **Theorem** The algorithms "random walk on spheres", "random walk on balls" and "random walk on lattices" of Monte Carlo methods are applicable to problems (30) and (31) [2], [3].

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[2] Smagulov Sh., Shakenov K.K., *Monte Carlo methods in hydrodynamics and filtrations*, Almaty, Publishing House "Kazakh University" (1999).

[3] Shakenov Kanat, *Numerical Modeling of the one Model of Filtration Process by Monte Carlo Methods* Series "Applied and Numerical Harmonic Analysis". Book "Methods of Fourier Analysis and Approximation Theory". Springer. Berkhäuser (2016), 237-258.

Bilinear estimates in Besov spaces generated by the Dirichlet Laplacian

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In this talk we consider the bilinear estimates in Besov spaces generated by the Dirichlet Laplacian, which were introduced on an open set in [1]. These estimates would be one of important tools to study the well-posedness for the initial-boundary value problem to nonlinear differential equations. These estimates are proved by the gradient estimates for heat semigroup together with the Bony paraproduct formula and boundedness of spectral multipliers for the Dirichlet Laplacian.

The talk is based on collaborations with Professors Tsukasa Iwabuchi of Tohoku University and Tokio Matsuyama of Chuo University.

[1] T. Iwabuchi, T. Matsuyama, and K. Taniguchi, *Besov spaces on open sets*, arXiv:1603.01334 (2016).

[2] T. Iwabuchi, T. Matsuyama, and K. Taniguchi, *Boundedness of spectral multipliers for Schrödinger operators on open sets*, to appear in Rev. Mat. Iberoam.

P-adic analysis

Organizers: A. Escassut, A. Khrennikov, K.O. Lindahl

p-Adic Model of Fluid in the Porous Medium

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Using a p-adic interpretation of a porous medium proposed in [1] and the hydrodynamic description of fluids in a porous medium given in [2], we develop a non-Archimedean mathematical model of fluid propagating through a porous medium. This is done in terms of a non-homogeneous Markov process, in the sense of [3]. We find the generator of that process.

The talk is based on collaborations with Andrei Khrennikov, Linnæus University, Sweden, and Anatoly N. Kochubei, Institute of Mathematics, Kiev, Ukraine.

[1] A. Khrennikov, K. Oleschko and M. J. Correa Lopez, *Application of p-adic wavelets to model reaction-diffusion dynamics in random porous media*, J. Fourier Anal. Appl. **22** (2016), 809-822.

[2] Shou D., Ye L., Fan J. *Treelike networks accelerating capillary flow*, Phys. Rev. E, **89** (2014), art. 053007.

[3] S. E. Kuznetsov, *Nonhomogeneous Markov processes*, Journal of Soviet Mathematics **25** (1984), 1380-1498.

Nevanlinna Theory outside of a hole

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Let K be an algebraically closed field of characteristic 0, complete with respect to an ultrametric absolute value. We establish a Nevanlinna Theory for meromorphic functions outside of a hole in K. We also give several applications of the theory to meromorphic functions outside a hole, such as on parametrization of elliptic and hyper-elliptic curves and results on branched values for p-adic meromorphic functions. Motzkin Factors, known for analytic elements, play here an essential role.

The talk is based on collaborations with Ta Thi Hoai An.

Applications of P-adic and ultrametric diffusion to model flows in capillary networks in porous media

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During the last 15 years p-adics has been actively applied to a variety of problems of theoretical physics (string theory, cosmology, quantum theory, disordered systems - spin glasses) and biology (modeling of the process of thinking and genetics). Recently [1] p-adics and especially theory of p-adic wavelets started to be applied to geophysics, e.g., to modeling of the diffusion dynamics in random porous media, e.g., flow of fluid (oil, water, oil-in-water or water-in-oil emulsions and droplets) in a complex network of pores with known topology. The present paper is of conceptual nature: we want to discuss in very general setting possibilities of applications of p-adics to geophysics, by using p-adic diffusion

representation of the master equations for transition of fluid between capillaries in porous media, and formulate a number of mathematical problems which have to be solved - to proceed further with such applications. We also stress the role of p-adic wavelets as the powerful tool to obtain analytic solutions of diffusion equations. Since p-adic diffusion is a special case of fractional diffusion and the latter closely coupled to the fractal structure of configuration space, p-adic geophyiscs can be considered as a new approach to fractal modeling of geophysical processes which is the well established area of research.
In spite of its mathematical beauty, the p-adic model does not reflect completely the complex branching structure of trees of capillaries in random porous media. Therefore the use of general ultrametric spaces is very important for concrete geological applications. Here we use theory of ultrametric wavelets and pseudo-differential operators which was established by S. Kozyrev and A. Khrennikov.

[1] A. Khrennikov, K. Oleschko and M. de Jesus Correa Lopez, *Modeling fluids dynamics with master equations in ultrametric spaces representing the treelike structure of capillary networks,* Entropy, **18** (2016), 249.

p-Adic Analogue of the Porous Medium Equation

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We consider a nonlinear evolution equation for complex-valued functions of a real positive time variable and a p-adic spatial variable. This equation is a non-Archimedean counterpart of the fractional porous medium equation. Developing, as a tool, an L^1-theory of Vladimirov's p-adic fractional differentiation operator, we prove m-accretivity of the appropriate nonlinear operator, thus obtaining the existence and uniqueness of a mild solution. We give also an example of an explicit solution of the p-adic porous medium equation.

The talk is based on collaborations with Andrei Khrennikov, Linnæus University, Sweden.

Linearization disks and geometric location of periodic points in p-adic and complex dynamics

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In this talk we discuss the relation between the geometric location of periodic points and local linearization of power series. In a recent paper [1] we use quadratic polynomials to show that periodic points are not the only obstruction for linearization in p-adic dynamics. In so doing we provide the first known examples in the dynamics of p-adic polynomials where the boundary of the linearization disk does not contain any periodic point. In fact, using a generalization by Kevin Keating [2] of Sen's Theorem [3] for lower ramification numbers of iterated power series, we localize all periodic points inside the open unit disc. We also discuss corresponding results from the complex field case due to work of Yoccoz, Perez-Marco, and Buff & Chéritat.

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Spectrum of ultrametric Banach algebras of bounded derivable functions

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Let K be an ultrametric complete field and let E be an open subset of K of strictly positive codiameter. Let $\mathcal{L}(E)$ be the Banach K-algebra of bounded derivable Lipschitzian functions from E to K and let $\mathcal{D}(E)$ be the Banach K-algebra of bounded strictly differentiable functions from E to K. It is shown that all elements of $\mathcal{D}(E)$ have a derivative that is continuous in E and that $\mathcal{D}(E)$ is strictly included in $\mathcal{L}(E)$. Given a positive number $r > 0$, all functions that are bounded and are analytic in all open disks of diameter r are strictly differentiable. Maximal ideals and continuous multiplicative semi-norms on $\mathcal{D}(E)$ and on $\mathcal{L}(E)$ are studied by recalling the relation of contiguity on ultrafilters: an equivalence relation. So, the maximal spectrum of $\mathcal{D}(E)$ (resp. $\mathcal{L}(E)$) is in bijection with the set of equivalence classes with respect to contiguity. Every prime ideal of $\mathcal{D}(E)$ (resp. $\mathcal{L}(E)$) is included in a unique maximal ideal and every prime closed ideal of $\mathcal{D}(E)$ (resp.$\mathcal{L}(E)$)) is a maximal ideal, hence every continuous multiplicative semi-norms on $\mathcal{D}(E)$ (resp. $\mathcal{L}(E)$) has a kernel that is a maximal ideal. If K is locally compact, every maximal ideal of $\mathcal{D}(E)$ (resp. $\mathcal{L}(E)$) is of codimension 1. Every maximal ideal of $\mathcal{D}(E)$ (resp. $\mathcal{L}(E)$) is the kernel of a unique continuous multiplicative semi-norm and every continuous multiplicative semi-norm is defined as the limit along an ultrafilter on E. Consequently, the set of continuous multiplicative semi-norms defined by points of E is dense in the whole set of all continuous multiplicative semi-norms. The Shilov boundary of $\mathcal{D}(E)$ (resp. $\mathcal{L}(E)$) is equal to the whole set of continuous multiplicative semi-norms. Many results are similar to those concerning algebras of uniformly continuous functions but some specific proofs are required.

The talk is based on collaborations with Alain Escassut.

On the Levi-Civita Fields: Introduction and Survey of Recent Research

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In this talk, I will introduce the Levi-Civita field $\mathcal{R}$, which is the smallest non-Archimedean ordered field extension of the real numbers that is real closed and complete in the order topology, and the complex field $\mathcal{C} := \mathcal{R} + i\mathcal{R}$. Then I will give a brief overview of selected topics of my work with my collaborators on $\mathcal{R}$ and $\mathcal{C}$ [1].
After reviewing the algebraic and topological structures of the fields [2], I will summarize the convergence and analytical properties of power series over $\mathcal{R}$ and $\mathcal{C}$, showing that they have the same smoothness behavior as real and complex power series [3]. Then I will present a Lebesgue-like measure and integration theory on $\mathcal{R}$ which has been recently extended to $\mathcal{R}^2$ and $\mathcal{R}^3$ [4].
A natural inner product can be defined on c_0, the space of null sequences of elements of $\mathcal{C}$, which induces the sup-norm of c_0. I will present characterizations of normal projections, adjoint and self-adjoint operators, and compact operators, and I will review the properties of positive operators on c_0 [5].

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On the bijective embedding of p-adic integers in the Cartesian product of p copies of the sets of 2-adic integers

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We study bijective embedding of p-adic integers in the Cartesian product of p copies of the sets of 2-adic integers. This embedding allows explicitly specify any p-adic integer through p specially selected 2-adic numbers. This representation can be used in p-adic Mathematical Physics, for example, in justifying choice of the parameter p.

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Special interest group: IGPDO Pseudo-differential operators

Organizers: S. Molahajloo, P. Wahlberg, M. W. Wong

Pseudo-diffential operators on $\mathbb{Z}$ related to a finite measure space

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Let $(S, \mathcal{B}, m)$ be a finite measure space and let $L^2(S, \mathcal{B}, m) = L^2(S)$ be the corresponding Hilbert space with the usual inner product $(\cdot,\cdot)_{L^2(S)}$. We suppose that there exists an orthonormal basis $\{e_n\}_{n\in\mathbb{Z}}$ for $L^2(S)$. The linear operator $\mathcal{F}_S : L^2(S) \to L^2(\mathbb{Z})$ defined by $(\mathcal{F}_S f)(n) =$
$= (m(S))^{-1/2}(f, e_n)_{L^2(S)}$, $n \in \mathbb{Z}$, for all f in $L^2(S)$ is a bijection such that $(\mathcal{F}_S f, \mathcal{F}_S g)_{L^2(S)} = (m(S))^{-1}(f, g)_{L^2(S)}$, for all f and g in $L^2(S)$.
Let $\sigma : \mathbb{Z} \times S \to \mathbb{C}$ be a measurable function. Then for every sequence a in $L^2(\mathbb{Z})$ we formally define the sequence $T_\sigma a$ by

$$(T_\sigma a)(n) = (m(S))^{-1/2} \int_S \sigma(n, s)(\mathcal{F}_{\mathbb{Z}} a)(s)\overline{e_n(s)}dm(s), \quad n \in \mathbb{Z}.$$

We call T_σ the pseudo-differential operator on $\mathbb{Z}$ (related to the measure space $(S, \mathcal{B}, m)$) corresponding to the symbol σ, whenever the integral exists for all $n \in \mathbb{Z}$. We can consider by duality the following two mathematical objects: $\mathcal{F}_{\mathbb{Z}} = \mathcal{F}_S^{-1} : L^2(\mathbb{Z}) \to L^2(S)$ defined by

$$(\mathcal{F}_{\mathbb{Z}} a)(s) = (m(S))^{1/2} \sum_{n=-\infty}^{\infty} a_n e_n(s), s \in S, a \in L^2(\mathbb{Z}) \quad \text{and}$$

$$(T_\tau f)(s) = (m(S))^{1/2} \sum_{n=-\infty}^{+\infty} \tau(s, n)\mathcal{F}_S(n)e_n(s), s \in S, f \in L^2(S),$$

where $\tau : S \times \mathbb{Z} \to \mathbb{C}$ is a measurable function. T_τ is called the pseudo-differential operator on S corresponding to the symbol τ. The aim of this talk is to study L^p-boundedness $(1 \le p < \infty)$, the Hilbert-Schmidt property and the compactness for the pseudo-differential operators defined above. When we take $S = S^1$, the unit circle centered at the origin, $dm(s) = ds$ and $e_n(s) = \frac{1}{\sqrt{2\pi}}e^{ins}$, for $s \in S^1$, $n \in \mathbb{Z}$, we recover the standard pseudo-differential operators on $\mathbb{Z}$ and S^1, which have been studied among others by Wong in his works.

Operator transformation of probability densities

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Using the operator methods of time-frequency analysis and quantum mechanics, we generalize the Edgeworth and Gram-Charlier series of standard probability theory. The main historical purpose of these series is to “correct” a Gaussian distribution when new information is given, such as moments, which does not agree with the Gaussian. These methods relate a probability density to a Gaussian by way of an operator transformation that is a function of the differentiation operator. We generalize the standard series in two ways. First, we relate any two probability densities using the differentiation operator. More generally, we show that any two probability densities may be related by an operator transformation, which is a function of *any* self-adjoint operator. We give a number of examples from various fields.

Solutions of a class of nonlinear stochastic equations on $\mathbb{R}^n$: A microlocal approach

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We study random field and function-valued solutions of certain hyperbolic stochastic partial differential equations, involving linear partial differential operators with polynomially bounded coefficients. We first analyse linear equations, and provide conditions on the initial data and on the stochastic terms, so that a random field solution exists and is unique. We then illustrate the case of some associated semilinear equations, and discuss an analogous existence and uniqueness result of a function-valued solution.

The talk is based on joint work with A. Ascanelli and A. Süss.

Time-frequency structure of the Green's function

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We consider the class of differential equations

$$\frac{d^n x(t)}{dt^n} + a_{n-1}\frac{d^{n-1}x(t)}{dt^{n-1}} + \ldots + a_1\frac{dx(t)}{dt} + a_0 x(t) = f(t) \tag{32}$$

where $a_0, \ldots, a_{n-1}$ are constant deterministic coefficients, $f(t)$ is the forcing term, and $x(t)$ is the solution. This class of equations represents a model for a wide variety of physical phenomena in several fields, such as electrical engineering, mechanical engineering, vibration of structures, thermodynamics, pharmacokinetics, and population genetics. A key design and analysis tool for (32) is the Green's function, obtained as its solution when the forcing term is a delta function. The frequency representation of the Green's function, defined as its Fourier transform, is the transfer function, another key tool for the study of phenomena modeled by (32). When the frequency spectrum of the forcing term changes with time, though, also the frequency spectrum of the solution changes with time. Time-frequency analysis is an extension of classical Fourier analysis which provides an effective description of time-varying frequencies [1], and the Wigner distribution is one the of the most common time-frequency distributions. We obtain the Wigner distribution of the Green's function of (32). Our result clarifies how this class of differential equation processes the time-varying frequencies of the forcing term to generate the time-varying frequencies of the solution. Surprisingly, the Wigner distribution of the impulse response is made by frequency translated replicas of a simple function of time and frequency, plus the interference terms due to the nonlinear nature of the Wigner distribution. We discuss this simple function and its properties, and we show how to reduce the interference terms by smoothing the Wigner distribution. Finally, we discuss an application of our result to system identification.

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Generalized microlocal elliptic pseudodifferential operators acting on $L^p_{\mathbf{loc}}(\Omega)$

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In the talk we introduce a class of vector weighted symbols $S_{m,\Lambda}(\Omega)$, where $m(\xi)$ and $\Lambda(\xi) = (\lambda_1(\xi), \dots, \lambda_n(\xi))$ are positive continuous weight functions and weight vectors. The corresponding pseudodifferential operators will be called m-pseudodifferential operators and they could be considered in the frame of general pseudodifferential calculus of R. Beals [1], Hörmander [3]. Thanks to the assumptions on the weight vector $\Lambda(\xi)$, the classes $S_{m,\Lambda}(\Omega)$ satisfy a condition of Taylor type and the m-pseudodifferential operators are L^p continuous.
In order to show microlocal properties of m-pseudodifferential operators the main problem consists in the complete lack of any homogeneity property of the weights $m(\xi)$ and $\Lambda(\xi)$. The characteristic set of $a(x,D)$ has not any conic properties with respect to the ξ variable. For this reason we introduce the concepts of characteristic filter of a m-pseudodifferential operator and of filter of weighted Sobolev regularity of $u \in \mathcal{D}'(\Omega)$. We can then obtain a result of microlocal propagation of singularities for solutions to (pseudo)differential equations. Some applications to linear partial differential operators are also given.

The talk is based on collaborations with Alessandro Morando, University of Brescia, Italy.

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Characterizations of nuclear pseudo-differential operators on $\mathbb{S}^1$

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In this paper, we give some necessary and sufficient conditions on the symbol of the pseudo-differential operator T_σ from $L^{p_1}(\mathbb{S}^1)$ into $L^{p_2}(\mathbb{S}^1)$ for $1 \le p_1, p_2 < \infty$, to be nuclear. We also show that the adjoint operator T_{σ^*}, from $L^{p_2'}(\mathbb{S}^1)$ into $L^{p_1'}(\mathbb{S}^1)$ is nuclear and present a formula for σ^* in the terms of σ. As application, a necessary and sufficient condition for self adjointness of nuclear pseudo-differential operators from $L^2(\mathbb{S}^1)$ into $L^2(\mathbb{S}^1)$ is given.

The heat kernel and trace class pseudo-differential operators on $\mathbb{R}^n$

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We use the heat kernel of the Laplacian on $\mathbb{R}^n$ and some fairly well-known results on nuclear operators to give a characterization of trace class pseudo-differential operators on $\mathbb{R}^n$. This gives another derivation of the trace formula for trace class pseudo-differential operators on $\mathbb{R}^n$.

This talk is based on collaborations with M. W. Wong. The research was carried out and completed during my visit of Professor M. W. Wong under the auspices of the International Visiting Research Traineeship (IVRT) in the Department of Mathematics and Statistics at York University.

Unified theory of the wavelet transformation of Frazier and Jawerth

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In 1990, Frazier and Jawerth [1] presented their well-known vast results for wavelet transformation of Sobolev spaces (and Besov and Lizorkin–Triebel spaces), using an operator approach leading to concise statements and ease of formulation. However, certain foundational questions for the synthesis operator (the inverse wavelet transform) were raised with only partial answers in the subsequent literature.
Recently it was shown in [2] how a unified theory of the synthesis operator can be build up from a precise definition of it, based on a Pettis integral. One main result is that two-sided invertibility of the wavelet transform is equivalent to biorthogonality of the chosen wavelets, as well as to the property that the wavelets form an unconditional basis of the space of temperate distributions (modulo polynomials). Nonetheless the terms in the synthesis operator can always be summed in any order—this virtue of the Pettis integral circumvents the dichotomy given by biorthogonality.
Moreover, in an analysis of Peetre's homogeneous Littlewood–Paley decomposition a few claims from the literature were corrected, and specific convergence rates were deduced for the wavelet decompositions. This will be explained as time permits, along with application to homogeneous Sobolev and more general Lizorkin–Triebel spaces with mixed norms.

The talk is based on work with Athanasios Georgiadis and Morten Nielsen.

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Spectral Invariance of Pseudo-differential Boundary Value Problems with Conical Singularities

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In this talk, we show how to prove the spectral invariance of the pseudo-differential calculus for boundary value problems with conical singularities on Lp spaces, as developed by B. W.- Schulze and E. Schrohe. We follow the standard argument of Gohberg and Hörmander. Nevertheless, due to the fact that we are working in the Lp-setting, we must deal with the Besov spaces on the boundary. As we will see, the argument can not be directly applied for these spaces, requiring some interesting modifications and better estimates. In this talk we will sketch the proof of the spectral invariance of the whole algebra, focusing on these necessary modifications that the Lp and Besov spaces require.

The talk is based on collaborations with Professor Elmar Schrohe.

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Pseudo-Differential Operators, Wigner Transforms and Weyl Transforms on the Poincaré Unit Disk

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Using the affine group and the Cayley transform from the unit disk $\mathbb{D}$ onto the upper half plane, we can turn $\mathbb{D}$ into a group, which we call the Poincaŕe unit disk. With this construction, $\mathbb{D}$ is a noncompact and nonunimodular Lie group. We characterize all infinite-dimensional unitary and irreducible representations of $\mathbb{D}$. By means of these representations, the Fourier transform on $\mathbb{D}$ is defined and the adjoint formula, the Plancherel theorem and hence the Fourier inversion formula, adjoint formula, Parseval identity and Plancherel theorem can be given. Then pseudo-differential operators with operator-valued symbols, operator-valued Wigner transforms, and Weyl transforms on $\mathbb{D}$ are defined.

The talk is based on collaborations with Professor M. W. Wong.

Factorizations and Singular Value Estimates of Operators with Gelfand-Shilov and Pilipovic kernels

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We prove that any linear operator with kernel in a Pilipović or Gelfand-Shilov space can be factorized by two operators in the same class. We apply these composition rules to deduce estimates of singular values and establish Schatten-von Neumann properties for such operators.

The talk is based on collaborations with Yuanyuan Chen and Joachim Toft.

Special interest group: IGCQA Quaternionic and clifford analysis

Organizers: S. Bernstein, U. Kähler, I. Sabadini, F. Sommen

Spin actions in Euclidean and Hermitian Clifford Analysis on Superspace

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In the previous work [1] we studied the group invariance of the inner product of supervectors as introduced in the framework of Clifford analysis on superspace. The fundamental group SO_0 of supermatrices leaving invariant such an inner product turns out to be an extension of $SO(m) \times Sp(2n)$. In this setting, superrotations (elements of SO_0) are no longer described by an even number of supervector reflections. That is why the spin group in superspace is introduced through the exponential of the so-called extended superbivectors. As usual, the spin group can be seen as a double covering of SO_0 by means of the representation $h(s)[x] = sx\overline{s}$.
In this talk, we study the invariance of the Dirac operator in superspace under the classical H and L actions of the spin group on superfunctions. Such actions give rise to the infinitesimal representation dL, $e^{dL} = L$. In the standard case, these operators correspond to the classical momentum operators (angular momentum plus spin). In addition, we study the invariance of the hermitian inner product of supervectors introduced in [2]. The group of complex supermatrices leaving it invariant constitutes an extension of $U(m) \times U(n)$ and is isomorphic to the subset SO_0^J of SO_0, of elements that commute with the complex structure J. The realization of SO_0^J within the spin group is studied together with the invariance under its actions of the twisted super Dirac operator $\partial_{J(x)}$.

The talk is based on collaborations with Hennie De Schepper and Frank Sommen.

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Stationary increment Gaussian processes and their derivatives: The quaternionic case

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We discuss in the quaternionic setting the relation between positive definite functions and the covariance of centered Gaussian processes, and on the construction of stochastic processes and their derivatives. We focus in particular on covariance functions of the form $r(t) + \overline{r(s)} - r(t-s)$, corresponding to stationary increment processes. The use of perfect spaces and strong algebras (introduced by the speaker and Guy Salomon, and which generalizes the notion of Banach algebra) as well as the notion of Fock space are crucial in this framework.

The talk is based on collaborations with Fabrizio Colombo, Irene Sabadini and Guy Salomon.

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A quadratic Fourier transform in Clifford analysis

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The Fourier transform is an important tool in mathematics and applications. Optically and physically motivated during the early 1970's in paraxial optics and quantum mechanics led to the linear canonical transform (LCT) ([1], [2]). The linear canonical transform has expanded into applied optics, mathematical physics and digital signal analysis.
Clifford analysis is a refinement of harmonic analysis and has proven to be useful to give a deeper insight in classical transforms. We consider the following quadratic Fourier transform

$$\mathcal{Q}f = \int_{\mathbb{R}^n} e^{iW(x,y)} f(y)\, dy,$$

where $W(x,y) = \frac{1}{2}db^{-1}\langle x,x\rangle - b^{-1}\langle x,y\rangle + \frac{1}{2}b^{-1}a\langle y,y\rangle$, and $\langle ., .\rangle$ denotes the ordinary scalar product in $\mathbb{R}^n$. The transform Q is isomophic to the Valen matrix

$$V = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

fulfilling

(i) $a, b, c, d \in \Gamma_n \cup \{0\}$ (Clifford group Γ_n)
(ii) $a\tilde{b}, \tilde{b}d, d\tilde{c}, \tilde{c}a \in \mathbb{R}^n$
(iii) $a\tilde{d} - b\tilde{c} = 1$,

where the reversion a is denoted by $\tilde{a}$. The transform is invertible. The Vahlen group [3] contains special subgroups, which describes special transforms. We will also highlight some applications.

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Landau's theorem for slice regular functions on the quaternionic unit ball

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During the development of the theory of *slice regular* functions over the real algebra of quaternions $\mathbb{H}$ in the last decade, some natural questions arose about slice regular functions on the open unit ball $\mathbb{B}$ in $\mathbb{H}$. We will present several new results in this context. More precisely we will start recalling the Schwarz–Pick lemma for slice regular functions $\mathbb{B} \to \mathbb{B}$ and then we will illustrate two variants of it, specialized to maps $\mathbb{B} \to \mathbb{B}$ that are not injective. These results allow a full generalization to quaternions of two theorems proven by Landau for holomorphic self-maps f of the complex unit disk with $f(0) = 0$. Landau had computed, in terms of $a := |f'(0)|$, a radius ρ such that f is injective at least in the disk $\Delta(0,\rho)$ and such that the inclusion $f(\Delta(0,\rho)) \supseteq \Delta(0,\rho^2)$ holds. We will describe in details the analogous result for slice regular functions $\mathbb{B} \to \mathbb{B}$ which allows a new approach to the study of Bloch–Landau-type properties of slice regular functions $\mathbb{B} \to \mathbb{H}$.

The talk is based on collaborations with C. Stoppato.

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Riemann-Hilbert problems for discrete monogenic functions

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Riemann-Hilbert problems (RH) are a basic tool in a wide range of applications, such as hydrodynamics or inverse scattering problems. While in the complex case connections between the continuous and discrete cases are well established (see, for instances, the study of discrete Riemann problems in connection to the 2D-Ising model) the same cannot be said about the higher dimensional case. For instance, while the analytic treatment of continuous RH-problems via Clifford analysis is well established in a variety of cases their discrete counterparts are missing. In fact, while discrete Clifford analysis started in the eighties and nineties with the construction of discrete Dirac operators for numerical methods of PDEs under the impulse of physical considerations the actual study of discrete boundary values started only recently. In this talk, we present the concept of discrete Hardy spaces as spaces of boundary values of discrete monogenic functions. Based on the concept of discrete Hardy spaces we study the solvability of discrete RH-problems in higher dimensions by constructing discrete Hardy decompositions. Furthermore, we are going to present results on the convergence of the discrete solutions to those of the continuous Riemann-Hilbert boundary value problem. In the end we will present connections with problems in image processing by looking at the concept of a discrete monogenic signal.

Generalizations of the the H^∞-functional calculus and applications to fractional diffusion processes

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The spectral theory based on the notion of S-spectrum, for quaternionic operators and in particular for vector operators, has been introduced about ten years ago and it is nowadays well developed. In this talk we show new results on the quaternionic version of the H^∞-functional calculus and we use it to compute the fractional powers of vector operators. It turned out that this theory allows to define new fractional diffusion processes.

Dirac operators for $SU(n)$ symmetries

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The Dirac equation arises from the linearization of a relativistic second order wave equation which describes the spin-$\frac{1}{2}$ electron by imposing $SU(2)$-symmetry. Nevertheless, several examples exist in which other type of symmetries is required. For instances, problems in optics often require $SU(3)$- symmetries, as in polarization, for example. In this talk we are interested in the problem of operators described by $SU(n)$-symmetries and modeled in terms of fractional derivatives.

Reconstruction of quaternionic signals with sparsity constraints

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Compressed Sensing is a new paradigm in signal processing which states that for certain matrices sparse representations can be obtained by a simple ℓ_1-minimization procedure. Here we explore this paradigm for higher-dimensional signals, in particular for quaternionic signals which represent color-encoded images such as RGB-images.

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On regular extension of quaternion functions

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A fundamental problem of a holomorphic extension of functions of one complex variable has also a great importance in applications. Some results are known about conditions for the holomorphic extension into a domain of function defined on a part of its boundary [1]. By means of the Kolosov-Muskhelishvili method these results can be used in nonclassical ill-posed problems of plane elasticity theory. In the three-dimensional theory of elasticity the situation is also similar. If stresses and displacements are given on a part of the boundary of a domain, then we have a three-dimensional ill–posed Cauchy problem for the elliptic Lame system. This problem can be reduced to the problem of the regular extension into a domain of the quaternion function defined on a part of its boundary. An advanced theory of regular (monogenic) quaternion functions and presence of various variants of the three-dimensional quaternion generalization of the Kolosov-Muskhelishvili formulae [2] can lead to new results in this area. In this report we discuss problems of the regular extension of quaternion functions and scopes of applications in the theory of elasticity.

This work was supported by grant 15-41-05081 from the Russian Foundation for Basic Research.

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A nonlinear boundary value problem with a shift for generalized hypermonogenic function in Clifford analysis

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In this talk, we discuss a nonlinear boundary value problem with a shift for generalized hypermonogenic function in Clifford Analysis. In the first part, we decompose the generalized hypermonogenic function into integral operators and discuss the properties of some integral operators. In the second part, by using the Plemelj formula for hypermonogenic functions and Schauder fixed point theorem, we prove the existence and uniqueness of the solution to the nonlinear boundary value problem with a shift for generalized hypermonogenic function.

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On a higher spin generalisation of the Fueter theorem

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The original Fueter theorem (1935) shows how we can construct functions $f : \mathbb{R}^4 \to \mathbb{H}$, that satisfy $Df := (\partial_0 + i\partial_i + j\partial_j + k\partial_k)f = 0$, starting from holomorphic functions. This theorem has been extended to the domain of Clifford analysis and can be used to construct monogenic functions (solutions of the Dirac operator) on $\mathbb{R}^m$ starting from, the easier to describe, holomorphic functions in the complex plane, see e.g. [2,4].
The aim of this talk is to introduce a higher spin version of Fueter's theorem and to do this we will start by giving an alternative proof for the classic theorem that lends itself well to generalisations. We will then introduce the setting in which we realise our higher spin framework, and provide the necessary tools for the higher spin Fueter theorem (i.e. a Fischer decomposition and an inversion).

The talk is based on collaborations with David Eelbode.

[1] Eelbode, D., Janssens, T., *Higher spin generalisation of Fueter's theorem.*
[2] Eelbode, D., Souček, V., Van Lancker, P., *Gegenbauer polynomials and the Fueter theorem*, Compl. Var. Ell. Equa. **59** No.6 (2014), p. 826-840.
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Interpolation in Clifford-Krein modules

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One of the central topics in hypercomplex function theory is interpolation. Interpolation by inner spherical monogenics provides the adequate counterpart to the classic finite Fourier transform and is widely used in applications. Interpolation problems for slice-hyperholomorphic functions which includes as particular cases the Nevanlinna-Pick and Carathéodory-Fejér interpolation problems were studied by D. Alpay, V. Bolotnikov, F. Colombo, and I. Sabadini. In more general terms, interpolation in hypercomplex function theory is closely linked to interpolation in reproducing kernel Hilbert spaces. But interestingly enough, in difference to the complex case there are many applications of hypercomplex function theory in which Hilbert spaces are not the natural setting. For instance, studying null-solutions of ultra-hyperbolic Dirac operators, pole figures of the crystallographic Radon transform which belong to the kernel of an ultra-hyperbolic Laplacian, or even more general problems in Minkowski space-time and string theory require inner product spaces with an indefinite inner product. Among other things this is due to the possibility of the underlying Clifford algebra to have a signature (p, q) and, therefore, to be linked to Pontryagin modules instead of Hilbert modules. As we shall see the natural setting for such problems is the Clifford-Krein module. After giving some general overview over Clifford-Krein modules with reproducing kernels we are going to study interpolation problems in these modules. Finally, we will present some applications of these problems.

The talk is based on collaborations with Daniel Alpay and Paula Cerejeiras.

On some properties of bicomplex modules endowed with a hyperbolic–valued norm

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In [1], there was introduced for the first time the notion of *hyperbolic–valued modulus* $|\cdot|_{\mathbf{k}}$. This modulus is defined on the set of bicomplex numbers $\mathbb{BC}$ and takes values in the set of non–negative hyperbolic numbers $\mathbb{D}^{+}$. In [2] there were developed many analytical and geometrical facts on the ring of bicomplex numbers and it was evident that the whole theory became quite rich when the hyperbolic–valued modulus $|\cdot|_{\mathbf{k}}$ goes into action.
The notion of hyperbolic–valued modulus gave rise to the notion of hyperbolic–valued norm on bicomplex moduli, see [1]. In this talk I will present further developments on the bicomplex functional analysis where the hyperbolic–valued modulus plays a key role. Finally we will see how the set of hyperbolic numbers behaves inside the bicomplex numbers in a quite similar way as the set of real numbers behaves inside the complex numbers and will comment on some geometric consequences of this fact.

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Hypermonogenic solutions and plane waves of the Dirac operator in $\mathbb{R}^p \times \mathbb{R}^q$

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In this talk we define monogenic solutions of the Dirac operator in $\mathbb{R}^p \times \mathbb{R}^q$ and find a Cauchy's integral formula for it in the unit sphere. Then we define the so called hypermonogenic plane wave solutions and compute some examples.

The talk is based on collaborations with Ali Guzman and Franciscus Sommen.

The unwinding Fourier decomposition method and related algorithms

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We will introduce a recently developed new signal processing method called unwinding Fourier decomposition. It is based on the front loading property of outer functions in the DSP theory and practice. Regarding the algorithmIn respect, in terms of Nevanlinna factorization of Hardy space functions we will introduce methods on how to extract the outer function part of a Hardy space function. We will promote a recently established method base on Szego's techniques.

The talk is based on collaborations with Lihui Tan.

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Some results of integral equations in real Clifford analysis

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By using the method of resolution of the identity, in this talk we define the generalized integrals in the sense of M. Spivak on open manifold for unbounded functions in real Clifford analysis, and discuss the solvability and the series expression of solutions for the second kind of generalized integral equations with Hölder continuous kernel and weakly singular kernel. By defining a new operator, we can overcome the non commutativity of the Clifford algebra. Finally we give the error estimate for the approximate calculation.

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Higher spin Laplace operator in several vector variables

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In this talk, I will introduce a class of conformallly invariant second order differential operators, known as higher spin Laplace operators. These must be seen as generalisations of the well-known Laplace operators in harmonic analysis, and the linearised Einstein equations.
We take a look at these operators from a function theoretical point of view, more specifically in the setting of harmonic analysis. The operators are differential operators acting on functions $f(x)$ on $\mathbb{R}^m$, taking values in the space of complex numbers $\mathbb{C}$. $\mathbb{C}$ is in fact a model for the irreducible representation of the orthogonal group $SO(m,\mathbb{C})$. We generalise the Laplace operator by replacing $\mathbb{C}$ with an arbitrary irreducible, finite dimensional $SO(m,\mathbb{C})$-representation, which has an integer highest weight $(l_1,\ldots,l_k,0,\ldots,0)$. Such representations can be modeled by special types of polynomial spaces, namely simplicial harmonic polynomials, making the function theoretical approach possible.
We will also take a look at a special type of polynomial solutions for the higher spin Laplace operators.

The talk is based on collaborations with Prof. David Eelbode (University of Antwerp) and Dr. Matthias Roels (University of Antwerp).

On the relations between monogenic and slice monogenic functions

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Monogenic functions with values in a Clifford algebra, namely functions in the kernel of the generalized Cauchy-Riemann operator, are widely studied in the literature. They are, in particular, harmonic functions in several variables. Slice monogenic functions (also called slice hyperholomorphic or slice regular functions) have been introduced in more recent times and they have applications, for example, in operator theory. It is thus a natural question to ask if there are relations between the two classes of functions.

The Fueter mapping theorem, which is a classical result in quaternionic and Clifford analysis (which has been investigated by several authors, at various degrees of generality) shows that to any slice monogenic function one may associate a monogenic function of axial type. Specifically, given f slice monogenic, we can represent in integral form the axially monogenic function $\breve{f}(x) = \Delta^{\frac{n-1}{2}} f(x)$ where Δ is the Laplace operator in dimension $n+1$, if $x \in \mathbb{R}^{n+1}$.
We have also solved the inverse problem: given an axially monogenic function $\breve{f}$ we can determine a slice monogenic function f (called Fueter's primitive of $\breve{f}$) such that $\breve{f}(x) = \Delta^{\frac{n-1}{2}} f(x)$. The proof of these results, in the case n even or odd is rather different.
In recent times, we have shown that by weakening a bit the definition of slice monogenic functions it is possible to prove that the Radon transform maps monogenic functions to slice monogenic functions with values in a Clifford algebra and, analogously, the dual Radon transform maps slice monogenic functions to monogenic functions. The proofs of these results are independent of the parity of the dimension.

Differential forms in Clifford superanalysis

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In our presentation we begin by defining the calculus framework for analysis in superspace starting with super-functions and super-distributions and then continuing with the main definitions for differential forms on superspace. We study the transformation of these differential forms under coordinates transformations thus showing that these objects can be defined in an invariant way. Next we investigate the topic of integration of differential forms starting with the integral of a differential form over a surface in Euclidean space. The problem however with differential forms is that there seems not to be an invariant way of defining the integral whereas for super-functions there is the invariant notion of the Berezin integral. The only possible setting for integration of differential forms seems to be via duality, which leads to the notion of a super-current. Super-surfaces may then be defined as special super-currents and involve delta distributions on superspace. In the next part of our lecture we extend the calculus of differential forms to Clifford superanalysis by combining it with the Clifford algebra. In particular, we introduce spin-invariant differential forms such as the differential of a vector variable and we study the calculus resulting from this. When comparing this calculus with the Cauchy integral formula on superspace it turns out that the theory of super-forms does not lead to the same Cauchy integral formula. This can be seen as a defect of the theory of differential forms on superspace that somehow seems to be unavoidable.

Gleason's problem associated to a ternary algebra

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We introduce of a new class of analytic functions defined on $\mathbb{T}$, a three dimensional structure different from $\mathbb{C} \times \mathbb{R}$, i.e. a commutative algebra given by the linear span of $\{1, e, e^2\}$, where $e \notin \mathbb{C}$ is a generating unit. We define a single ternary conjugate and we build our analytic theory differently than in the existing literature, on the basis of this single conjugation (akin to the quaternionic case). We give the solution to the Gleason problem which gives rise to Fueter–type variables and study related problems.

The talk is based on collaborations with D. Alpay and M.B. Vajiac.

Bernstein type Inequalities for Multicomplex Polynomials

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In this talk we consider the well–known Bernstein and Erdős–Lax inequalities in the case of bicomplex polynomials. We shall prove that the validity of these inequalities depends on the norm in use and we consider the cases of the Euclidean, Lie, dual Lie and hyperbolic–valued norms. In particular, we show that in the case of the Euclidean norm the inequalities holds keeping the same relation with the degree of the polynomial that holds in the classical complex case, but we have to enlarge the radius of the ball. In the case of the dual Lie norm both the relation with the degree and the radius of the ball have to be changed. Finally, we prove that the exact analogs of the two inequalities hold when considering the Lie norm and the hyperbolic–valued norm. In the case of these two norms we also show the validity of the Maximum Modulus Principle for bicomplex holomorphic functions. We also show the extension of some of these results to the multicomplex case.

The talk is based on collaborations with I. Sabadini and A. Vajiac.

Special interest group: IGPDE Recent progress in evolution equations

Organizers: M. D'Abbicco, M. R. Ebert, M. Reissig

Critical exponents for two semilinear evolution equations with structural damping

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In this talk, we discuss the critical exponents $p_0 = 1 + 2\sigma/(n - 2\sigma)$ and $p_1 = 1 + 2\delta/n$ for the two semilinear evolution equations

$$u_{tt} + (-\Delta)^{\delta} u_t + (-\Delta)^{\sigma} u = \begin{cases} |u|^p \\ |u_t|^p \end{cases}$$

with $\sigma, \delta \in \mathbb{N} \setminus \{0\}$, and $2\delta \leq \sigma$, for $t \geq 0$ and $x \in \mathbb{R}^n$. We show the global existence of small data solutions in the supercritical ranges, and that each one of the critical exponent is related to one of the two asymptotic profiles of the solution to the linear problem in $L^1 - L^q$ setting, $q \in [1, \infty]$. These two profiles are respectively given by the solutions to

$$v_t + (-\Delta)^{\kappa} v = 0, \qquad t \geq 0, \; x \in \mathbb{R}^n,$$

with suitable initial data, with $\kappa = \sigma - \delta$ if $n(1 - 1/q) > 2\delta$, or $\kappa = \delta$ if $n(1 - 1/q) < 2\delta$. The exponents p_0 and p_1 are sharp, in the sense that no global solution to the nonlinear problem may exist in the subcritical and critical range, under a sign assumption on the data.

The talk is based on collaborations with Marcelo R. Ebert (University of São Paulo).

[1] M. D'Abbicco, M.R. Ebert, *A new phenomenon in the critical exponent for structurally damped semi-linear evolution equations*, Nonlinear Analysis **149** (2017), 1–40.

The Dirichlet problem and boundary regularity for nonlinear parabolic equations

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The p-parabolic equation

$$\partial_t u = \Delta_p u := \operatorname{div}(|\nabla u|^{p-2} \nabla u)$$

is a nonlinear cousin of the classical heat equation. As such, it offers both difficulties and advantages compared with the heat equation. In the talk, we consider the Perron method for solving the Dirichlet problem for the p-parabolic equation in general bounded domains in $\mathbf{R}^{n+1}$. Compared to space-time cylinders, such domains allow the space domain to change in time.

Of particular interest will be boundary regularity for such domains, i.e. whether solutions attain their boundary data in a continuous way. Relations between regular boundary points and barriers will be discussed, as well as some peculiar examples and surprising phenomena related to boundary regularity.

If time permits, the so-called normalized p-parabolic equation, with surprisingly different behaviour, will also be discussed.

The talk is based on collaborations with Anders Björn (Linköping), Ugo Gianazza (Pavia) and Mikko Parviainen (Jyväskylä).

[1] A. Björn, J. Björn, U. Gianazza and M. Parviainen, *Boundary regularity for degenerate and singular parabolic equations*, Calc. Var. Partial Differential Equations **52** (2015), 797-827, `arXiv:1310.6845`

[2] A. Björn, J. Björn and U. Gianazza, *The Petrovskiĭ criterion and barriers for degenerate and singular p-parabolic equations*, to appear in Math. Ann., doi:10.1007/s00208-016-1415-0, `arXiv:1509.02279v2`

Well-Posedness for Schrödinger Operators with Time Dependent Hamiltonian

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We consider Schrödinger operators with time dependent hamiltonian that may vanish at some point, say at $t = 0$. We discuss the optimal relation between the order of vanishing and the decay of the drift term as the space variable x tends to infinity. We provide necessary and sufficient conditions for the well-posedness of the Cauchy problem in Sobolev, Gevrey and Gelfand-Shilov spaces.

Backward parabolic equations with non Lipschitz coefficients

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Consider the equation

$$Pu = \partial_t u + \sum_{i,j} \partial_{x_i}(a_{i,j}(t,x)\partial_{x_j} u) + \sum_j b_j(t,x) u \partial_{x_j} + c(t,x)u = 0$$

on the strip $[0,T] \times \mathbb{R}^n_x$. The coefficients are supposed to be real valued, measurable and bounded. The matrix $(a_{jk})_{j,k=1,\dots,n}$ is symmetric and positive definite.
It is well known that the Cauchy problem for this equation, when the data are given on $\{t = 0\}$, is an ill-posed problem. Nevertheless a result due essentially to Hurd says that:
for every $T < 0$ and for every $T' \in]0,T[$ and $D > 0$ there exist $\rho > 0$, $0 < \delta < 1$ and $M > 0$ such that, if $u \in \mathcal{E}$ is a solution of $Pu \equiv 0$ on $[0,T]$ with $\|u(0,\cdot)\|_{L^2} \leq \rho$ and $\|u(t,\cdot)\|_{L^2} \leq D$ on $[0,T]$, then

$$\sup_{t\in[0,T']} \|u(t,\cdot)\|_{L^2} \leq M \|u(0,\cdot)\|^{\delta}_{L^2},$$

where $\mathcal{E} := C^0([0,T], L^2(\mathbb{R}^n)) \cap C^0([0,T[, H^1(\mathbb{R}^n)) \cap C^1([0,T[, L^2(\mathbb{R}^n))$.
The constants ρ, M and δ depend only on T, T' and D, on the ellipticity constant of P, on the L^∞ norms of the coefficients $a_{i,j}$'s, b_i's, c and of their spatial derivatives, and on the Lipschitz constant of the coefficients $a_{i,j}$'s with respect to time.
In this communication I will consider the situation in the case that the coefficients are non Lipschitz-continuous.

The talk is based on collaborations with Daniele Casagrande (University of Udine) and Martino Prizzi (University of Trieste).

Semi-linear systems of weakly coupled damped waves

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Let us consider the following model of semi-linear system of weakly coupled classical waves

$$\begin{cases} u_{tt} - \Delta u + \frac{1}{(1+t)^{r_1}} u_t = f(v), t \in [0,\infty), x \in \mathbb{R}^n \\ v_{tt} - \Delta v + \frac{1}{(1+t)^{r_2}} v_t = g(u), t \in [0,\infty), x \in \mathbb{R}^n \\ (u, u_t, v, v_t)(0,x) = (u_0, u_1, v_0, v_1)(x). \end{cases}$$

where $r_1, r_2 \in (0,1)$, and for $[s-1]^+ = \max\{s-1, 0\}$ the data

$$(u_0, u_1) \in (H^{s_1} \cap L^m) \times (H^{[s_1-1]^+} \cap L^m),$$

$$(v_0, v_1) \in (H^{s_2} \cap L^m) \times (H^{[s_2-1]^+} \cap L^m),$$

We consider several cases with respect to the regularity of the data. Our aim is to investigate the global existence of solutions for small initial data.

Joint work with Prof. Michael Reissig.

Theory of wave models with integrable and decaying in time speed of propagation

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In this work we discuss about the Cauchy problem for damped wave equations with a time-dependent propagation speed and dissipation. The model of interest is

$$u_{tt} - a(t)^2 \Delta u + b(t) u_t = 0, \;\; u(0,x) = u_0(x), \;\; u_t(0,x) = u_1(x).$$

Let $a \in \mathcal{C}^2 \cap L^1$ be a strictly positive function and $b \in \mathcal{C}^1$ be a real-valued function. Then we propose a classification of dissipation terms in non-effective and effective. In each case we derive estimates for kinetic and elastic type energies by developing a suitable WKB analysis. Moreover, we show optimality of results by the aid of scale-invariant models and we explain by an example that in some estimates a loss of regularity appears. Finally, our interest is to apply the derived linear estimates to study global existence of small data solution to semi-linear models.

The talk is based on joint works with Michael Reissig (TU Bergakademie Freiberg).

[1] M. R. Ebert, M. Reissig, *Regularity theory and global existence of small data solutions to semi-linear de Sitter models with power non-linearity*, 32 A4, submitted.

[2] M. R. Ebert, M. Reissig, *Theory of damped wave models with integrable and decaying in time speed of propagation,* Journal of Hyperbolic Differential Equations, **13** (2016), 417–439.

On the two-dimensional Zakharov–Kuznetsov equation

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Zakharov–Kuznetsov equation

$$u_t + bu_x + u_{xxx} + u_{xyy} + uu_x = 0$$

is considered on a rectangle $\Sigma = (0,R)\times(0,L)$. Besides initial condition $u\big|_{t=0} = u_0(x,y)$, boundary conditions $u\big|_{x=0} = \mu_0(t,y)$, $u\big|_{x=R} = \nu_0(t,y)$, $u_x\big|_{x=R} = \nu_1(t,y)$ are set as well as boundary conditions on the sides $y=0$ and $y=L$, which can be homogeneous Dirichlet, homogeneous Neumann or periodic. Consider, for example, Neumann case $u_y\big|_{y=0} = u_y\big|_{y=L} = 0$.
Results on global well-posedness in the classes of weak and regular solutions as well as on large-time decay of small solutions are established. Let $T>0$ be arbitrary, $B_T = (0,T)\times(0,L)$.
Theorem 1. Let $u_0 \in L_2(\Sigma)$, $\mu_0,\nu_0 \in H^{s/3,s}_{t,y}(B_T)$ for $s > 3/2$, $\nu_1 \in L_2(B_T)$, $\mu_{0y}(t,0) = \nu_{0y}(t,0) = \mu_{0y}(t,L) = \nu_{0y}(t,L) \equiv 0$. Then the problem is well-posed in the space $C_w([0,T];L_2(\Sigma)) \cap L_2(0,T;H^1(\Sigma))$.
Theorem 2. Let $u_0 \in H^3(\Sigma)$, $\mu_0,\nu_0 \in H^{4/3,4}_{t,y}(B_T)$, $\nu_1 \in H^{1,3}_{t,y}(B_T)$, $u_{0y}(x,0) = u_{0y}(x,L) \equiv 0$, $\mu_0(0,y) \equiv u_0(0,y)$, $\nu_0(0,y) \equiv u_0(R,y)$, $\nu_1(0,y) \equiv u_{0x}(R,y)$, $\partial_y^{2j+1}\mu_0(t,0) = \partial_y^{2j+1}\nu_0(t,0) = \partial_y^{2j+1}\nu_1(t,0) = \partial_y^{2j+1}\mu_0(t,L) = \partial_y^{2j+1}\nu_0(t,L) = \partial_y^{2j+1}\nu_1(t,L) \equiv 0$ for $j=0$ and $j=1$. Then the problem is well-posed in the space $\{\partial_t^m u \in C([0,T];H^{3(1-m)}(\Sigma)) \cap L_2(0,T;H^{3(1-m)+1}(\Sigma)), m \le 1\}$.
Theorem 3. Let $u_0 \in L_2(\Sigma)$, $\mu_0 = \nu_0 = \nu_1 \equiv 0$, $bR^2 < 3\pi^2$, then there exist $\epsilon_0 > 0$ and $\kappa > 0$, such that if $\|u_0\|_{L_2(\Sigma)} \le \epsilon_0$, the corresponding unique solution to the considered problem $u \in C_w([0,T];L_2(\Sigma)) \cap L_2(0,T;H^1(\Sigma))$ $\forall T>0$ satisfies an inequality

$$\|u(t,\cdot,\cdot)\|_{L_2(\Sigma)} \le \sqrt{1+R}e^{-\kappa t}\|u_0\|_{L_2(\Sigma)} \quad \forall\ t>0.$$

Similar results are obtained for the problems set on the whole strip $\mathbb{R}\times(0,L)$ and on the half-strip $\mathbb{R}_+\times(0,L)$.

Finite lifespan of solutions of the semilinear wave equation in the Einstein-de Sitter spacetime

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We present some results on the semilinear massless waves propagating in the Einstein-de Sitter spacetime. The equation for the self-interacting massless field in the Einstein-de Sitter spacetime is

$$-\partial_t^2\psi - 2\partial_t\psi + t^{-\frac{4}{3}}A(x,\partial_x)\psi = \lambda|\psi|^{p-1}\psi\,, \tag{33}$$

where $A(x,\partial_x)$ is a second order partial differential operator. It has singular coefficient and belongs to the family of the non-Fuchsian partial differential operators. The initial value problem for this equation with the data on hyperplane $t=0$ must be defined properly. In [1] the weighted initial value problem for the covariant wave equation was introduced and explicit representation formulas for the solutions were obtained. In [2] the parametrices in terms of Fourier integral operators were constructed and the propagation and reflection of singularities phenomena were discussed. In [1] and [2] the operator $A(x,\partial_x)$ is the Laplace operator.
In this talk, we present the generalization of the setting introduced in [1] for equation (33). We show that for $1<p<4$ the solution of the massless self-interacting scalar field equation in the Einstein-de Sitter universe has finite lifespan [3].

The talk is based on collaboration with Karen Yagdjian.

[1] Anahit Galstian, Tamotu Kinoshita, Karen Yagdjian, *A note on wave equation in Einstein and de Sitter space-time*, J. Math. Phys. **51** (2010).

[2] Anahit Galstian, Karen Yagdjian, *Microlocal analysis for waves propagating in Einstein & de Sitter spacetime*, Math. Phys. Anal. Geom. **17** (2014), 223–246.

[3] Anahit Galstian, Karen Yagdjian, *Finite lifespan of solutions of the semilinear wave equation in the in Einstein-de Sitter spacetime*, arXiv: 1612.09536 (2017).

On the classification of the spectrally stable standing waves of the Hartree problem

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We consider the fractional Hartree model, with general power non-linearity and space dimension. We construct variationally the "normalized" solutions for the corresponding Choquard-Pekar model - in particular a number of key properties, like smoothness and bell-shapedness are established. As a consequence of the construction, we show that these solitons are spectrally stable as solutions to the time-dependent Hartree model.
In addition, we analyze the spectral stability of the Moroz-Van Schaftingen solitons (see [1]) of the classical Hartree problem, in any dimensions and power non-linearity. A full classification is obtained, the main conclusion of which is that only and exactly the "normalized" solutions (which exist only in a portion of the range) are spectrally stable.

The talk is based on collaboration with Atanas Stefanov (University of Kansas, USA).

[1] V. Moroz, J. Van Schaftingen, *Ground states of nonlinear Choquard equations: existence, qualitative properties and decay asymptotics*, J. Funct. Anal. **265** (2013), no. 2, 153–184.

Exact decay rate of solutions to second order evolution equations

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We consider an abstract second order evolution equation with damping of the form

$$u'' + u' + Au + f(u) = 0.$$

The linear operator A is self-adjoint and nonnegative, with a discrete spectrum, and the nonlinear term $f(u)$ has order greater than one at the origin. For such equations there is the coexistence of slow solutions and fast solutions. Slow solutions live close to the kernel of A, and decay as negative powers of t as solutions of the first order equation obtained by neglecting the operator A and the second order time-derivatives in the original equation. Fast solutions live close to the range of A and decay exponentially as solutions of the linear homogeneous equation obtained by neglecting the nonlinear term in the original equation. The abstract results apply to semilinear dissipative hyperbolic equations.

The talk is based on collaborations with Massimo Gobbino and Alain Haraux.

[1] M. Ghisi, M. Gobbino, A. Haraux, *Optimal decay estimates for the general solution to a class of semi-linear dissipative hyperbolic equations*, J. Eur. Math. Soc. (JEMS) **18** (2016), 1961-1982.

[2] M. Ghisi, M. Gobbino, A. Haraux, *Finding the exact decay rate of all solutions to some second order evolution equations with dissipation*, Journal of Functional Analysis **271** (2016), 2359-2395.

Quantization of energy and weak turbulence for some second order evolution equations with nonlinear damping

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We consider the second order evolution equation

$$u''(t)+|u'(t)|^2u'(t)+Au(t)=0,$$

where A is a self-adjoint operator with dense domain on a Hilbert space H. The spectrum of A is an increasing and diverging sequence of positive eigenvalues. The damping term depends on a power of the *norm* of the velocity.
It can be easily shown that, as $t\to+\infty$, all nonzero solutions to this equation decay to 0 as $t^{-1/2}$ in the phase space $D(A^{1/2})\times H$. This leads us to investigate the asymptotic behavior of the rescaled variable $\sqrt{t}\cdot u(t)$.
We find a rather unexpected dichotomy phenomenon. Solutions with finitely many Fourier components are asymptotic to solutions of the non-dissipative linearized equation $v''(t)+Av(t)=0$, and exhibit some sort of equipartition of the energy among the components. Solutions with infinitely many Fourier components tend to zero weakly but not strongly (weak turbulence).
We show also that the limit of the energy of solutions depends only on the number of their Fourier components (quantization of energy).

The talk is based on joint work with Marina Ghisi (Pisa) and Alain Haraux (Paris VI).

[1] M. Ghisi, M. Gobbino, A. Haraux. *Quantization of energy and weakly turbulent profiles of the solutions to some damped second order evolution equations*, arXiv:1701.08604

On the well-posedness for second order hyperbolic equations with time dependent oscillating coefficients

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We consider the well-posedness for the Cauchy problem of second order hyperbolic equation with time dependent coefficients:

$$\begin{cases}\left(\partial_t^2-a(t)^2\partial_x^2+2b(t)\partial_x\partial_t\right)u(t,x)=0, & (t,x)\in(0,T]\times\mathbb{R},\\ u(T,x)=u_0(x),\quad u_t(T,x)=u_1(x), & x\in\mathbb{R}.\end{cases}\tag{P}$$

In particular, we are interested in the case that the characteristic roots vanish at $t=0$. If a and b are polynomial, then the necessary and sufficient conditions for the well-posedness of (P) in a suitable class, for instance C^∞, Gevrey class, etc., are described by the orders of a and b as $t\to0$ (cf. [1]). However, the "orders" of the coefficients are not necessarily essential if a and b are oscillating functions. Indeed, there exist examples of a and b such that (P) must not be C^∞ well-poesed from a viewpoint of the order, but actually (P) is C^∞ well-poed (cf. [2]). In this talk we shall develop the results in [2] taking account of some smoothness properties to a and b, which was studied in [3].

[1] T. B. N. Bui, F. Hirosawa, *On energy estimates for second order hyperbolic equations with Levi conditions for higher order regularity*, Ann. Univ. Ferrara Sez. VII Sci. Mat. **57** (2011), 317-339.

[2] F. Hirosawa, M. Reissig, *Levi condition for hyperbolic equations with oscillating coefficients*, J. Differential Equations **223** (2006), 329-350.

[3] V.Ya. Ivrii, V.M. Petkov, *Necessary conditions for the Cauchy problem for non-strictly hyperbolic equations to be well-posed*, Uspekhi Mat. Nauk **29** (1974) 3-70 (in Russian) English translation: Russ. Math. Surveys **29** (1974), 1-70.

A Generalized Levi Condition for Weakly Hyperbolic Cauchy Problems with Coefficients Low-Regular in Time

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We consider weakly hyperbolic equations of the form

$$\begin{cases} D_t^m u = \sum_{j=1}^{m} \sum_{|\gamma|=j} a_{j,\gamma}(t)\lambda(t)^j D_x^\gamma D_t^{m-j} u + \sum_{|\gamma|<j} b_{j,\gamma}(t) D_x^\gamma D_t^{m-j} u, \\ D_t^{k-1} u(0,x) = g_k(x), \quad (t,x) \in [0,T] \times \mathbb{R}^n,\ k = 1, \dots, m, \end{cases}$$

whose characteristic roots $\tau_k = \tau_k(t, \xi)$ satisfy the separation condition

$$|\tau_j(t, \xi) - \tau_k(t, \xi)| \ge \delta\lambda(t)\langle\xi\rangle, \quad j \ne k, \text{ for all } \xi \in \mathbb{R}^n,\ t \in [0, T],$$

where $\lambda = \lambda(t)$ is a shape function with $\lambda(0) = \lambda'(0) = 0$. It is well known that (in general) we have to impose Levi conditions to get C^∞ or Gevrey well-posedness, even if the coefficients are smooth with respect to time.
In this talk, we discuss our approach to this problem when the coefficients are not smooth but only μ-continuous, where μ is a modulus of continuity. We introduce a generalized Levi condition which yields well-posedness of the above problem in weighted spaces $H^\nu_{\eta,\delta} = \{u \in \mathcal{S}' : e^{\delta\eta(\langle D_x \rangle)} u \in H^\nu\}$, where the weight function η depends on the shape function λ as well as on the modulus of continuity μ.
We discuss some examples that link our work to existing results for Levi conditions for weakly hyperbolic equations and to results for strictly hyperbolic equations with low-regular coefficients. The talk is concluded by looking at the case of coefficients that also depend on x.

The talk is based on collaborations with Michael Reissig (TU Bergakademie Freiberg).

Nonlinear evolution equations and its applications to chemotaxis models

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In this talk we show the existence of the solution and asymptotic behavior for nonlinear evolution equations(cf [2]):

$$(NE) \begin{cases} u_{tt} = D\Delta u_t + \nabla \cdot (\chi(u_t, e^{-u}) e^{-u} \nabla u) + \mu(1 - u_t)u_t & \text{in } \Omega \times (0,T) \\ \frac{\partial}{\partial \nu} u|_{\partial\Omega} = 0 & \text{on } \partial\Omega \times (0,T) \\ u(x,0) = u_0(x),\ u_t(x,0) = u_1(x) & \text{in } \Omega \end{cases}$$

- $u := u(x,t), \quad \text{for}(x,t) \in \Omega \times (0,T)$

- Ω is a bounded domain in R^n with a smooth boundary $\partial\Omega$ and ν is a unit outer normal vector
- $\chi(s_1, s_2)$ is sufficiently smooth, D and μ are positive constants.

Then we apply the result to a degenerate chemotaxis model:

$$(CM)\begin{cases} u_t - \Delta u = -\mathrm{div}(\chi u \nabla v) + \mu_1 u(1 - u - a_1 v) \\ v_t = \mu_2 v(1 - a_2 u - v) \\ u, v \text{ satisfy initial-0-flux boundary value condition} \end{cases}$$

where $\mu_1, \mu_2 > 0$, $\chi > 0$. For $|a_1|, |a_2| < 1$, (CM) is considered in [1].

The talk is based on collaborations with J. I. Tello

[1] Akisato Kubo, Jose Ignacio Tello, *Mathematical analysis of a model of chemotaxis with competition terms*, Differential and Intergral Equations, **29(5-6)** (2016), 441-454.

[2]Akisato Kubo, Hiroki Hoshino, *Nonlinear evolution equations with strong dissipation and proliferation*, Current Trends in Analysis and its Applications, Birkhäuser (2015), 233-241.

Focusing nonlinear Klein Gordon equation with potential

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We describe the dynamics for the focusing nonlinear Klein-Gordon equation

$$u_{tt} - \Delta u + m^2 u = V(x)|u|^{p-1}u$$

with positive radial potential V and initial data in the energy space. Under suitable assumption on the potential, we establish the existence and uniqueness of ground state solution that enables us to define a treshold size for the initial data that separates global existence and blow-up. We find a global existence result for subcritical exponents and subcritical energy data. For subcritical exponents and critical energy some solutions blow-up, other solutions exist for all time according to suitable decomposition of the energy space of the initial data into two complementary domains determined by the sign of the conformal energy functional which involves the potential V .

The talk is based on collaborations with Vladimir Georgiev.

[1] Kenig, C. E., Merle, F., *Global well-posedness, scattering and blow-up for the energy-critical focusing non-linear wave equation.* Acta Mathematica, **201** (2008), 147 – 212.

Gevrey well posedness of the generalized Goursat-Darboux problem for linear PDEs with constant coefficients

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We consider the generalized Goursat-Darboux problem for third order linear PDE's with constant coefficients. Our purpose is to find necessary conditions for this problem to be well-posed in the Gevrey classes Γ^s with $s > 1$. It is proved that there exists some critical index s_0 such that if the Goursat-Darboux problem is well posed in Γ^s for $s > s_0$ then the coefficients of the derivatives with respect to one of the variables are zero. In order to prove our results, we first construct an explicit solution of a family of problems with data depending on a parameter $\eta > 0$ and then we obtain an asymptotic representation of a solution as η tends to infinity.

The talk is based on collaborations with Prof. Jaime Carvalho e Silva.

[1] Bronshtein, M. D., Necessary solvability conditions for the Goursat problem over Gevrey spaces, *Izv. VUZ. Matem.*, **21 (8)** (1977) 19-30.

[2] Carvalho e Silva, J. and Leal, C., The generalized Goursat-Darboux problem for a third order operator, *Proc. Am. Math. Soc.*, **125 (2)** (1997) 471-475.

[3] Carvalho e Silva, J. and Srivastava, H. M., Asymptotic representations for a class of multiple hypergeometric functions with a dominant variable, *Integral transforms and special functions*, **11 (2)** (2001) 137-150.

[4] Hasegawa, Y., On the C^∞ Goursat problem for 2nd order equations with real constant coefficients, *Proc. Japan Acad.*, **51 (7)**, (1975) 516–519.

[5] Rodino, L., *Linear partial differential operators in Gevrey spaces*, Singapore: World Scientific (1993).

The regularity of the semilinear term on the Cauchy problem for the Schrödinger equation

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Some modified Strichartz estimates are constructed and applied to the Cauchy problem for the critical semilinear Schrödinger equation in the Sobolev space of fractional order. Small global solutions are obtained under less regularity assumption for the semilinear term.

The talk is based on collaborations with Professor Takeshi Wada of Shimane University.

[1] H. Pecher, *Solutions of semilinear Schrödinger equations in H^s*, Ann. Inst. H. Poincaré Phys. Théor. **67** (1997), no. 3, 259–296.

[2] H. Uchizono, T. Wada, *On well-posedness for nonlinear Schrödinger equations with power nonlinearity in fractional order Sobolev spaces*, J. Math. Anal. Appl. **395** (2012), no. 1, 56–62.

A classification for wave models with time-dependent mass and speed of propagation

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In this talk we will consider the Cauchy problem for a wave equation with time-dependent propagation speed and mass

$$\begin{cases} u_{tt} - a(t)^2\Delta u + m(t)^2 u = 0, \quad (t,x) \in (0,\infty)\times\mathbb{R}^n, \\ (u(0,x), u_t(0,x)) = (u_0(x), u_1(x)), \quad x \in \mathbb{R}^n. \end{cases} \tag{34}$$

We plan to propose a classification for the mass term $m(t)^2u$ that should depend on the speed of propagation $a(t)$. We will consider the case $a \notin L^1$ and we expect that the case $a \in L^1$ shall be treated using a different approach, see [1] and [2]. The classification should be a natural generalization of the one proposesd in [3] and [4] in the case of constant speed of propagation $a(t) \equiv 1$.

The talk is based on collaborations with Prof. Dr. Marcelo Ebert.

[1] Bui Tang Bao Ngoc, M. Reissig *The interplay between time-dependent speed of propagation and dissipation in wave models*. Eds. M. Ruzhansky and V. Turunen, Fourier analysis, Trends in Mathematics, Birkhauser, (2014), 9-45.
[2] M. R. Ebert, M. Reissig, *Theory of damped wave models with integrable and decaying in time speed of propagation,* Journal of Hyperbolic Differential Equations, v. 13 (2016), 417-439.
[3] C. Böhme, *Decay rates and scattering states for wave models with time-dependent potential,* Ph.D. Thesis, TU Bergakademie Freiberg, 2011, 143pp.
[4] W. N. Nascimento, *Klein-Gordon models with non-effective*, PhD thesis, UFSCar (2016) 179 pp.

Fractional chain rule and application to a semi-linear scale-invariant wave equation with damping and mass

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In the talk we discuss the fractional chain rule and its application to the Cauchy problem for a semi-linear wave equation with scale-invariant damping and mass terms and with power non-linearity.
The model of interest is

$$\begin{cases} u_{tt} - \Delta u + \frac{\mu_1}{1+t} u_t + \frac{\mu_2^2}{(1+t)^2} u = |u|^p, \\ u(0,x) = u_0(x), \ u_t(0,x) = u_1(x), \end{cases}$$

where μ_1, μ_2^2 are non-negative constants.
We consider the global (in time) existence of small data solutions. In particular, we study the interplay between the power p, coefficients μ_1, μ_2^2, the data spaces regularity and spaces for solutions.
Moreover, we show how it is possible to apply some inequalities from harmonic analysis to the case in which we assume $H^\sigma \times H^{\sigma-1}$, $\sigma > 1$ regularity for initial data.

The results of this talk are obtained in collaboration with Michael Reissig (TU Bergakademie Freiberg).

$L^p - L^q$ decay estimates for the linear fractional diffusive equation

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In this talk we discuss the $L^p - L^q$ decay estimates for the solution of fractional diffusive equation

$$\begin{cases} \partial_t^{1+\alpha} u - \Delta u = 0, \qquad t \geq 0, \ x \in \mathbb{R}^n, \\ u(0,x) = u_0(x), \\ u_t(0,x) = u_1(x), \end{cases} \tag{35}$$

where $\alpha \in (0,1)$ and $\partial_t^{1+\alpha} u$ is the Caputo fractional derivative in time defined by $\partial_t^{1+\alpha} u(t,x) := J^{1-\alpha}(\partial_t^2 u)(t,x)$ and J^β is the Riemann-Liouville fractional integral, given by

$$J^\beta f(t) = \frac{1}{\Gamma(\beta)} \int_0^t (t-s)^{\beta-1} f(s)\, ds,$$

for $0 < \beta < 1$ and Γ is the Euler gamma function. As application we present some results to the semilinear problem associated to (1).

The talk is based on collaborations with Marcelo Ebert (University of São Paulo - Brazil) and Marcello D'Abbicco (University of Bari - Italy).

Regularity theory and global existence of small data solutions to semi-linear de Sitter models with power non-linearity

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In the talk we discuss the Cauchy problem for semi-linear de Sitter models with power non-linearity. The model of interest is

$$\phi_{tt} - e^{-2t}\Delta\phi + n\phi_t + m^2\phi = |\phi|^p, \quad (\phi(0,x), \phi_t(0,x)) = (f(x), g(x)),$$

where m^2 is a non-negative constant and n is the dimension. We study the global (in time) existence of small data solutions. In particular, we show the interplay between the power p, admissible data spaces and admissible spaces of solutions (Sobolev solutions, energy solutions or classical solutions).

The results of this talk are obtained in collaboration with Marcelo Rempel Ebert (University of Sao Paulo).

Large time behavior of solutions to strong damped wave equations

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We derive the higher order expansion formulas for the solutions to the Cauchy problem of the strong damped wave equation

$$\partial_t^2 u - \Delta u - \nu\Delta\partial_t u = 0 \text{ in } (0,\infty) \times \mathbb{R}^n,$$

with $u(0,x) = u_0(x)$, $\partial_t u(0,x) = u_1(x)$, where $\nu > 0$. We apply these estimates to prove the second order expansion formulas of the global solution for semilinear strong damped wave equations with nonlinearity $f(u)$ like $|u|^{p-1}u$, $p > 1$. We note that the contribution of u_0 to the asymptotic profiles firstly appears from the second order expansion.

The talk is partially based on collaborations with Professor Ryo Ikehata (Hiroshima university).

Asymptotic behavior of solutions to the wave equation with space-dependent damping

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We consider the initial-boundary value problem for the damped wave equation

$$\begin{cases} \partial_t^2 u - \Delta u + a(x)\partial_t u = 0, & x \in \Omega, t > 0, \\ u(x,t) = 0, & x \in \partial\Omega, t > 0, \\ u(x,0) = u_0(x),\ \partial_t u(x,0) = u_1(x), & x \in \Omega \end{cases}$$

in an exterior domain $\Omega \subset \mathbb{R}^N$ $(N \geq 2)$. Our purpose is to study how the damping term influences the asymptotic behavior of the solution when the coefficient depends on the space variable. In particular, as a typical case, we assume that

$$a(x) \sim |x|^{-\alpha} \quad (|x| \to \infty)$$

with some $\alpha \in \mathbb{R}$, and we investigate the effect of the parameter α in the energy decay and asymptotic behavior of the solution. In this talk, we derive almost sharp decay estimates of the energy of the solution when $\alpha < 1$. Moreover, we prove that the asymptotic profile of the solution is given by a solution of the corresponding parabolic problem.

The talk is based on collaborations with Motohiro Sobajima (Tokyo University of Science).

[1] M. Sobajima, Y. Wakasugi, *Diffusion phenomena for the wave equation with space-dependent damping in an exterior domain*, J. Differential Equations **261** (2016), 5690–5718.

[2] M. Sobajima and Y. Wakasugi, *Remarks on an elliptic problem arising in weighted energy estimates for wave equation with space-dependent damping term in an exterior domain*, AIMS Mathematics, 2017, **2** (1): 1–15.

Asymptotic profiles of solutions for the linearized damped extensible beam equation with variable coeffients

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We study a one-dimensional beam equation with variable coefficients. From the physical point of view the equation arises from linearized equation of e.g. the Woinowsky-Krieger equation describing extensible beam (see [1]). From the mathematical viewpoint, it corresponds to linearized Kirchhoff equation with 4th order space-derivative term. Our aim of this study is to classify the property of solutions by the decay rate of given variable coeffieicents with respect to time.

The talk is based on collaborations with Yuta Wakasugi (Ehime University).

[1] Reinhard Racke and Shuji Yoshikawa, *Decay estimates for the Cauchy problem for the damped extensible beam equation*, Applicable Analysis, **95** (2016), 1118-1136.

Integral transform approach to the Klein-Gordon equation of quantum field theory in curved spacetime

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In this talk we present an integral transform that allows to write solutions of the partial differential equation with variable coefficients via solutions of simpler equation. (For details, see [1].) The application of this transform to the issue of global in time existence of the solution to the Klein-Gordon equation of quantum field theory in some curved spacetimes will be discussed. In particular, we will present a completion of the results of [2] on the global in time existence of solution of the Klein-Gordon equation in the de Sitter spacetime for the self-interacting scalar field.

[1] Karen Yagdjian, *Integral transform approach to solving Klein-Gordon equation with variable coefficients*, Mathematische Nachrichten, **288** (17-18)(2015), 2129-2152.

[2] Anahit Galstian, Karen Yagdjian, *Global in Time Existence of the Self-Interacting Scalar Field in De Sitter Spacetimes*, Nonlinear Anal. Real World Appl. **34** (2017), 110–139.

Special interest group: IGGF Special session on generalized functions and applications

Organizers: M. Kunzinger, M. Oberguggenberger, S. Pilipović

Abstract Friedrichs systems and universal operator extension

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The Friedrichs theory [4] of positive symmetric systems of first order partial differential equations encompasses many standard equations of mathematical physics, irrespective of their type. More recently this theory was recast in an abstract Hilbert space setting [3, 1].
We make a further step, presenting a purely operator theoretical description of abstract Friedrichs systems, and proving that any pair of abstract Friedrichs operators admits bijective extensions with a signed boundary map. Moreover, we provide sufficient and necessary conditions for existence of infinitely many such pairs of spaces, and by the universal classification theory (see [5]) we get a complete identification of all such pairs, which we illustrate on a simle example.

The talk is based on collaborations with Marko Erceg and Alessandro Michelangeli.

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Directional short-time Fourier transform and directional regularity

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We prove the continuity results for the directional short-time Fourier transform (DSTFT) and the corresponding synthesis operator on test function spaces $\mathcal{K}_1(\mathbb{R}^n)$ and $\mathcal{D}(\mathbb{S}^{n-1})\widehat{\otimes}\mathcal{K}_1(\mathbb{R})\widehat{\otimes}\mathcal{U}(\mathbb{C}^n)$, respectively, as well as on their duals. Then, we introduce and analyze the directional wave front for tempered and exponential distributions.

The talk is based on collaborations with professor Stevan Pilipović and professor Katerina Saneva.

Microlocal regularity of linear partial differential operators with generalized coefficients

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The notion of regularity in the Colombeau algebra $\mathcal{G}(\Omega)$ is based on the subalgebra $\mathcal{G}^{\infty}(\Omega)$ which plays the same role as $\mathcal{C}^{\infty}(\Omega)$ in $\mathcal{D}'(\Omega)$, and it is the basis of the development of local and microlocal analysis within $\mathcal{G}(\Omega)$, see [3,6,7]. However, the $\mathcal{G}^{\infty}-$regularity does not exhaust the regularity problem inherent to the algebra $\mathcal{G}(\Omega)$. Given a set of sequences of real numbers $\mathcal{R}$, a sheaf of subalgebras $\mathcal{G}^{\mathcal{R}}(\Omega)$ of $\mathcal{G}(\Omega)$ defines a new notion of local $\mathcal{R}-$regularity for generalized functions of $\mathcal{G}(\Omega)$; the microlocalization of this $\mathcal{R}-$regularity has also been done, see [2] and [1]. The aim of this work is to tackle the problem of $\mathcal{R}-$microlocal regularity of solutions of linear partial differential equations with $\mathcal{R}-$regular functions as coefficients in the spirit of the works of [4] and [5].

The talk is based on collaboration with T. Saidi.

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Fujita type blow-up for discrete reaction-diffusion equations on networks

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This talk is concerned with long time behaviors of solutions to the reaction-diffusion equations $u_t = \Delta_\omega u + \psi(t)|u|^{q-1}u$ with the non-trivial and nonnegaive initial data. The propose of this talk is to introduce a critical set $C(\psi)$ in the following sense:
(i) solutions blow up in finite time, for $q \in C(\psi)$.
(ii) solutions with small initial data are exponentially decreasing, for $q \notin C(\psi)$.
In order to prove the main theorems, we first derive the comparison principles for the solutions of the equation above, which play an important role throughout our work. In addition, we give some numerical illustrations which exploit the main results.

A new blow-up condition for solutions to p-Laplacian parabolic equations

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This lecture is concerned with a non-existence(blow-up) of the (weak) solutions to the initial and boundary value problem

$$(E)\begin{cases} u_t = \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right) + f(x,u), & in\ \ \Omega\times(0,T),\\ u(\cdot,0)=u_0, & in\ \ \Omega,\\ u=0, & on\ \ \partial\Omega\times(0,T), \end{cases}$$

where $p \geq 2$ and $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$.
More precisely, we prove that the solutions to (E) blow up in finite time, for sufficiently large initial data $L^\infty(\Omega)\cap H_0^p(\Omega)$, under a condition for source term $f \geq 0$ that

$$(C_p) \qquad \alpha\int_0^u f(s)\,ds \leq uf(u) + \beta u^p + \gamma,\ \ u>0,$$

for some α, β, and $\gamma > 0$ with $0 < \beta \leq \frac{(\alpha-p)\lambda_0}{p}$, where λ_0 is the first eigenvalue of the p-Laplace operator $\Delta_p u := div\left(|\nabla u|^{p-2}\nabla u\right)$. Moreover, it will be shown that the condition (C_p) improves the blow-up conditions ever known so far (for example, see the conditions in [1], [2], and [3]).

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Convolutors in spaces of tempered ultradistributions

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In this talk we study convolutors in spaces of tempered ultradistributions, both of Beurling and Roumieu type. Our first main objective is to construct a test function space whose dual is algebraically isomorphic to the space of convolutors. We achieve this goal by characterizing the space of convolutors in terms of the short-time Fourier transform. Secondly, we discuss the locally convex structure of the space of convolutors (endowed with the operator topology) and show that the above isomorphism holds topologically in the Beurling case while it does not hold topologically in the Roumieu case.

The talk is based on collaboration with Jasson Vindas.

Recent results on the Fatou-Riesz Tauberian theorem

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The Fatou-Riesz Tauberian theorem is one of the most important Tauberian theorems. In one version, convenient for our purposes, it says that if a real-valued function τ, defined on the positive half-axis, satisfies the Tauberian condition *slowly decreasing*, i.e. for each $\varepsilon > 0$, there is $\delta > 0$ such that

$$\liminf_{x\to\infty} \inf_{h\in[0,\delta]} \tau(x+h) - \tau(x) \geq -\varepsilon, \tag{36}$$

and its Laplace transform

$$\mathcal{L}\{\tau; s\} = \int_0^\infty \tau(x)e^{-sx}\mathrm{d}x \quad \text{converges for } \Re e\, s > 0 \tag{37}$$

and admits a continuous extension on the line $\Re e\, s = 0$, then

$$\tau(x) = o(1), \quad x \to \infty. \tag{38}$$

A natural question one could pose in regard to this theorem is how much the boundary behavior of $\mathcal{L}\{\tau; s\}$ can be weakened, while still retaining the conclusion (38). In this talk, we will weaken it to a minimum. Namely, the boundary behavior we will impose is *local pseudofunction* boundary behavior and it is minimal in the sense that (38) implies (36), (37) and that $\mathcal{L}\{\tau; s\}$ admits local pseudofunction boundary behavior on the line $\Re e\, s = 0$. We will also discuss further related results we obtained. Most of the material that will be presented is available in [1].

The talk is based on collaborations with Jasson Vindas.

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A Picard-Lindelöf theorem for singular nonlinear PDE

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We present a Picard-Lindelöf theorem for normal singular nonlinear PDE

$$\begin{cases} \partial_t^k y = F\left(t, x, \left(\partial_t^j \partial_x^\alpha y\right)_{\substack{j<k \\ |\alpha|+j\leq k}}\right) \\ \partial_t^j y(0,x) = f_j(x) & 0 \leq j < k \end{cases}$$

where F and f_j can be, e.g., Schwartz distributions. The proof is based on a generalization of the Banach fixed point theorem, but with loss of derivatives, without the uniqueness part, and starting the iterates exactly from the initial condition. To realize this goal, some necessary requirements needed to be satisfied. Free composition of generalized functions (GF) is one of those. Since that's not simple if we define GF as functionals, we followed Cauchy and Dirac and thought GF as smooth set-theoretical functions that can take infinite values and derivatives. But in that case, the Lipschitz constants could be infinite; so we needed a language with infinitesimal and infinite numbers. But if we have an infinitesimal h, then the ODE $y'(1+y)h = -t$ has solution defined only in the infinitesimal interval $[-\sqrt{h}, +\sqrt{h}]$. Therefore, we also needed GF defined only on infinitesimal sets. Finally, to define Fréchet-like spaces of GF, we needed a good notion of compact set for GF and for their norms. *Generalized smooth functions* are a minimal branch of Colombeau's theory that allows for all these possibilities, see [1], and the result is hence proved in this framework. We will see that this result applies to a vast class of PDE, e.g. if F is a polynomial and f_j are Dirac deltas, but it does not apply to equations such as $\partial_t y = \delta(y)$, $y(0,x) = 0$ because the Lipschitz condition fails.

The talk is based on collaborations with Lorenzo Luperi Baglini.

See at my home page http://www.mat.univie.ac.at/~giordap7:

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[2] Giordano P., Luperi Baglini L., Picard-Lindelöf theorem for PDE.

The generalized Hankel-Clifford transformation on certain spaces for a class of tempered ultradistributions

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The present paper aims to study theory for generalized Hankel-Clifford transformation on certain spaces of generalized functions. To extend the transform to a space of tempered ultradistributions, the class of rapid descent ultra-differentiable functions are stated. Mappings involving various differential operators are investigated and stated to be continuous. The theory developed is applied to solve some partial differential equations involving generalized Kepsinki-type-operator with tempered ultra-distributional initial conditions.

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The Picard-Lindelöf method in Colombeau type algebras

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We extend earlier preliminary results concerning a convenient Picard–Lindelöf–Cauchy–Lipschitz theorem in the setting of algebras of Colombeau type generalized functions. The main tool is a fixed point theorem which goes beyond the known results of contractions in ultrametric spaces, which are not "fine" enough for our purpose. Using this fixed point theorem and an algebra with asymptotics defined by the differential problem, we are able to establish the announced theorem on existence and uniqueness of solutions to the ODE with irregular data. In view of going beyond this, we report on our progress in tackling the problem of the transport equation, a partial differential equation, with irregular coefficients. We also try to compare our results to those of other authors and approaches, in particular those in the framework of generalized smooth functions (GSF).

The talk is based on joint work with Jean-André Marti.

Generalized functions and electromagnetic fields

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We discuss formulations for electromagnetic fields corresponding to or acting on charged particles in terms of distributional tensor fields or $U(1)$-bundle curvature. The generic non-smoothness in combination with the non-linearity of the interaction requires a careful mathematical set-up and methods from nonlinear theories of generalized functions, in particular regularization approaches and non-smooth differential geometry, will be employed.

The talk is based on collaboration with Christian Spreitzer.

On the convolution of ultradistributions of Beurling and Roumieu type with spiral supports

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We prove some theorems concerning existence and various prpperties of the convolution in the spaces of ultradistributions of Beurling and Roumieu type as well as in the spaces of tempered ultradistributions of Beurling and Roumieu type, respectively, under new conditions for their supports.
We discuss various situations when supports of considered ultradistributions are contained in a spiral type subsets of $\mathbb{R}^d$. They lead to wider classes of compatible and polynomially compatible supports which are unbounded in each direction of $\mathbb{R}^d$.

The talk is based on a joint work with Svetlana Mincheva-Kamińska.

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Distributed order fractional constitutive stress-strain relation in a model of the wave equation

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Recently, the classical wave equation has been generalized for the case of viscoelastic materials by the use of fractional derivatives of constant real order (cf. [1,2]). In this work we use distributed order fractional model to describe wave propagation in viscoelastic infinite media, and study existence and uniqueness of fundamental solutions for the corresponding generalized Cauchy problem. Some particular cases of distributed order fractional constitutive stress-strain relations are examined in details, and numerical experiments are presented to illustrate theoretical results.

The talk is based on collaborations with Ljubica Oparnica and Dušan Zorica.

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Inverse function theorems for generalized smooth functions

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Generalized smooth functions (GSF) are an extension of Colombeau's theory of nonlinear generalized functions. They provide a non-Archimedean framework in which many results from classical analysis can be lifted to generalized functions, often with proofs that closely resemble the standard ones. In this talk we present a number of local and global inverse function theorems in GSF and discuss their relation to other approaches, in particular to the discontinuous Colombeau differential calculus of Aragona, Fernandez, Juriaans and Oberguggenberger.

This is joint work with Paolo Giordano.

Optimal control problems in algebras of generalized functions

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This talk is devoted to the study of optimal control problems for finite dimensional nonlinear systems with constraints in algebras of generalized functions. Particularly, we consider optimal control problems with the cost functional

$$J(u) = \phi(t_1, y(t_1)) + \int_{t_0}^{t_1} L(t, y, u)\, dt, \tag{39}$$

which is to be minimized with respect to a nonlinear state system

$$\partial_t y = F\big(t, y, u\big), \quad y(t_0) = y^0, \tag{40}$$

with additional constraints, where the state $y(t) \in \mathbb{R}^n$ and the control $u(t) \in \mathbb{R}^m$, $t_0 \leq t \leq t_1$. The study includes optimal control problems with distributional initial data, with state equations which contain singular (delta) potentials and problems with functionals which might be generalized functions.
We interpret the constrained minimization problems as variational problems with constraints in algebras of generalized functions and derive necessary conditions for the optimal pair.

The talk is based on collaboration with Michael Oberguggenberger.

Modeling of generalized stochastic problems

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We distinguish two basic approaches in modeling stochastic problems. The first is modeling stochastic equations for random processes on the basis of deterministic laws of physics, biology, economics, etc. in combination with describing the effects of random perturbations. The second one is the derivation of deterministic PDEs for various probabilistic characteristics of the simulated processes, in particular, infinite-dimensional extensions of Chapman–Kolmogorov and Fokker–Planck equations.
In accordance with modern challenges of stochastic theory and applications we pay special attention to modeling infinite-dimensional stochastic Cauchy problems. On the basis of the first approach for processes depending on spatial variables $x \in \mathbb{R}^n$, at the beginning we obtain the stochastic Cauchy problem with Ito integral with respect to perturbations in the form of Brownian sheet $W(x,t), t \geq 0$. Then, after generalized differentiation in x, we obtain the problem with a cylindrical Wiener process, which should be understood in a generalized sense with respect to x and time variable t.
It remains unclear whether the cylindrical Wiener process is a characteristic feature of the models considered or some changes (which?) in the formulation can reduce to equations with Q-Wiener processes.
As for the second approach, in contrast to classical results on relations between stochastic equations and corresponding PDEs obtained under rather restrictive conditions, that do not take place in applications, we obtain generalized PDEs for stochastic characteristics under conditions consistent with applications.

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On integrability and convolvability of ultradistributions of Roumieu type via approximate units

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We consider several general sequential conditions for convolvability of ultradistributions of Roumieu type in the space $\mathcal{D}'^{\{M_p\}}(\mathbb{R}^d)$ and prove their equivalence. The discussed conditions, based on two classes of approximate units and a result on integrability of ultradistributions of Roumieu type, are analogous to the known convolvability conditions in the spaces $\mathcal{D}'$ of distributions and $\mathcal{D}'^{(M_p)}$ of ultradistributions of Beurling type. Moreover, the following property of the convolution and ultradifferential operator $P(D)$ of class $\{M_p\}$ is proved: if $S, T \in \mathcal{D}'^{\{M_p\}}(\mathbb{R}^d)$ are convolvable, then

$$P(D)(S * T) = (P(D)S) * T = S * (P(D)T).$$

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Anisotropic distributions and applications

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H-distributions were introduced by Antonić and Mitrović (2011) as an extension of H-measures to the $\mathrm{L}^p - \mathrm{L}^q$ setting. Their variants have been successfully applied to problems in velocity averaging (Lazar and Mitrović 2012) and compensated compactness with variable coefficients (Mišur and Mitrović 2015). Unlike H-measures, which are nonnegative Radon measures, H-distributions are only distributions in the Schwartz sense, which follows from the standard Schwartz kernel theorem.
To give a precise description of H-distributions, we will introduce the notion of anisotropic distributions – distributions of different order with respect to different coordinate directions. In order to show that H-distributions are anisotropic distributions of finite order with respect to every coordinate direction, we will prove a variant of the Schwartz kernel theorem.
We will use our variant of the Schwartz kernel theorem to show a generalisation of the Peetre theorem on coordinate-free characterisation of partial differential operators, as well.

The talk is based on collaboration with Nenad Antonić and Marko Erceg.

The uniform moment asymptotic expansion

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In the span of several articles, of which most results are nicely gathered in [1], R. Estrada and R. P. Kanwal introduced and studied the properties of the so called Moment Asymptotic Expansion of distributions, which expresses the asymptotic, for the topology of $\mathcal{D}'$, of a certain distribution f as

$$f(\lambda x) \sim \sum_{n=0}^{\infty} \frac{(-1)^n \mu_n \delta^{(n)}(x)}{n! \lambda^{n+1}}, \qquad \text{as } \lambda \to \infty,$$

for certain numbers μ_n called the *moments* of f. In this talk, we extend this notion into the realm of ultradistributions, an discover its natural uniform nature. The ultimate goal is to give a complete characterisation and discuss possible applications.

The talk is based on collaborations with Jasson Vindas.

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Colombeau algebras without asymptotics

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All constructions of Colombeau algebras so far incorporate certain asymptotic estimates for the definition of the spaces of moderate and negligible functions, the quotient of which forms the algebra. There is a certain degree of freedom in the asymptotic scale employed for these estimates; most commonly a polynomial scale is used, but there exist generalizations in several directions. Moreover, these estimates commonly employ test objects which also can be chosen in many different ways.
I will present a (diffeomorphism invariant, full) algebra of generalized functions which, instead of asymptotic estimates obtained by inserting appropriate test objects, employs only topological estimates on certain spaces of kernels for its definition. This is a direct generalization of the usual seminorm estimates valid for distributions and appears to be a promising concept for the further development of nonlinear generalized function spaces. Moreover, this algebra is almost universal in the sense that there are canonical morphisms from it into most of the classical Colombeau algebras.

Stochastic Fourier integral operators in linear elasticity

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The talk is part of a project aiming at representing wave propagation in random media by means of stochastic Fourier integral operators. Waves in a linear, homogeneous, isotropic medium are described by the (3×3)-system $\rho_0 \partial_t^2 u = \mu \Delta u + (\lambda + \mu)\nabla(\operatorname{div} u) + f$ for the displacement $u = (u_1, u_2, u_3)$ with the constant density ρ_0 and the Lamé constants λ, μ. Writing $u = \nabla\varphi + \operatorname{curl}\vec{\psi}$ with $\operatorname{div}\vec{\psi} = 0$ reduces the system to four uncoupled wave equations for the potentials φ, $\vec{\psi}$ of the form $\partial_t^2 \varphi = c^2 \Delta\varphi + \chi$ (and similarly for $\vec{\psi}$) [2]. Splitting the wave operators in products of two half wave operators $\partial_t \pm \mathrm{i}c\sqrt{-\Delta}$, the solution $u(x,t)$ can finally be synthesized from Fourier integral operator blocks of the form

$$b(x,t) = \iiint \mathrm{e}^{\mathrm{i}(x\xi - c|\xi|t)} \int_0^t \mathrm{e}^{\mathrm{i}c|\xi|s} \widehat{\chi}(\xi, s)\, \mathrm{d}s\mathrm{d}\xi.$$

Based on this mean field representation, random properties of the medium can be modelled by means of perturbed phase functions $(x\xi - c(x)|\xi|t)$ in the integrands, where $c(x)$ is a random perturbation of the mean propagation velocity c. The coefficient $c(x)$ is chosen as a random field with shape parameters calibrated by measured waves. The Fourier integral operators are evaluated numerically using the cost saving butterfly algorithm [1]. In this way, a confidence bound around the mean field solution can be computed. The ultimate goal of this research is to provide a method for damage detection in elastic bodies from the time-dependent response induced by point-source excitations.

The talk reports on joint work with Martin Schwarz (Innsbruck).

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Soliton dynamics for the Degasperis-Procesi equation

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The Degasperis-Procesi equation is, in fact, the family of third order dispersive conservation laws,

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left\{c_0 u + c_1 u^2\right\} = \varepsilon^2 \frac{\partial}{\partial x}\left\{\alpha \frac{\partial^2 u}{\partial t \partial x} - \gamma \frac{\partial^2 u}{\partial x^2} + c_2 \left(\frac{\partial u}{\partial x}\right)^2 + c_3 u \frac{\partial^2 u}{\partial x^2}\right\},$$

where $x \in R^1$, $t > 0$; $c_i \geq 0$, $\alpha \geq 0$, and $\gamma \geq 0$ are constants, and $\varepsilon > 0$ characterizes the dispersion value. This family contains only three integrable equations: 1. the KdV equation ($\alpha = c_2 = c_3 = 0$), 2. the Camassa-Holm equation ($\gamma = 0$, $c_1 = -3c_3/2\alpha^2$, $c_2 = c_3/2$), and 3. the Degasperis-Procesi equation by itself ($\gamma = 0$, $c_1 = -2c_3/\alpha^2$, $c_2 = c_3$). It is known also that the KdV equation has soliton as an exact traveling wave solution, the Degasperis-Procesi equation has a non-smooth analog of soliton called "peakon" whereas the Camassa-Holm equation has, depending on c_0 either solitons or peakons.
The aim of the talk is the following: 1. To consider the separation of the Degasperis-Procesi family (of the set of the constants c_i, α, γ) into clusters of equations with soliton and peakon traveling wave solutions respectively, and 2. To discuss the scenario of soliton collision for essentially non integrable cases. Our main tool is the Weak Asymptotic Method [1,2].

The talk is based on collaborations with my student J. Noyola Rodriguez.

[1] V.G. Danilov, G.A. Omel'yanov, *Weak asymptotics method and the interaction of infinitely narrow delta-solitons*, Nonlinear Analysis: Theory, Methods and Applications, **54** (2003), 773–799.
[2] G. Omel'yanov, *Multi-soliton collision for essentially nonintegrable equations*, In: Generalized Functions and Fourier Analysis, Series: Operator Theory: Advances and Applications, **260**, (2017), 153-170.

Weyl asymptotic formulas for infinite order ΨDOs and Sobolev type spaces. Part II: Heat kernel analysis; Infinite order Sobolev type spaces

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This is the second in a series of three talks concerning a class of infinite order pseudo-differential operators. I will present: a) The heat kernel analysis needed for the proofs of the Weyl asymptotic formulae for the class of operators. The main goal is the semigroup $T(t)f = \sum_{j=0}^{\infty} e^{-t\lambda_j}(f, \varphi_j)\varphi_j$, $f \in L^2(\mathbb{R}^d)$, $t \geq 0$, with infinitesimal generator $-\overline{A}$ (the closure of $-a^w$ in $L^2(\mathbb{R}^d)$) where λ_j and φ_j are the eigenvalues and eigenfunctions of $\overline{A}$.

b) Infinite order Sobolev type spaces $H^*_{A_p,\rho}(f)$, where the order is given by a functions f belonging to a certain class of "admissible" functions of sub-exponential (i.e. ultrapolynomial) growth. $H^*_{A_p,\rho}(f)$ satisfies most of the familiar results for the classical, finite order, Sobolev spaces. Moreover, I will present the Fredholm properties of infinite order ΨDOs having hypoelliptic symbols satisfying elliptic bounds with respect to an admissible function f

The talk is based on collaborative works with Marco Cappiello, Bojan Prangoski and Jasson Vindas.

Weyl asymptotic formulae and Sobolev spaces for infinite order pseudo-differential operators. Part I: hypoellipticity, semi-boundedness, the heat kernel

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This is the first in a series of two talks concerning a class of infinite order pseudo-differential operators. The corresponding symbols are global and of Shubin type and the natural functional framework that arises are the Komatsu spaces of non-quasi-analytic tempered ultradistributions. Here we discuss the symbol classes and the calculus that they enjoy as well as the appropriate notion of hypoellipticity and the construction of parametrices. At last, we will talk about realisation in $L^2(\mathbb{R}^d)$ of such hypoelliptic operators, their semi-boundedness when the symbols additionally are real-valued and positive and, at the very end, we give precise estimates on the heat kernel which are instrumental for deriving the asymptotics of the eigenvalue counting function (the main goal of the second talk in this series given by Stevan Pilipović).

The talk is based on collaborations with Stevan Pilipović and Jasson Vindas.

Frame expansions of tempered distributions and ultradistributions

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In this talk we present expansions in a Fréchet space and its dual via localized frames. In particular, we obtain frame expansions in the Schwartz space and the space of tempered distributions, as well as frame expansions of tempered ultradistributions. Furthermore, we extend a known characterization of the Schwartz space and its dual, based on the Hermite basis, to a characterization based on a larger class of frame functions. A step further, we introduce a more general concept of "localization" of Fréchet frames and investigate the corresponding frame-related operators.

The talk is based on collaborations with Stevan Pilipović.

Degenerate C-distribution semigroups and C-(ultra)distribution cosine functions in locally convex spaces

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The main topic of this talk are degenerate C-distribution semigroups and degenerate C-(ultra)distribution cosine functions in the setting of barreled sequentially complete locally convex spaces. In our approach, the infinitesimal generator of a degenerate C-(ultra)distribution cosine function is a multivalued linear operator and the regularizing operator C is not necessarily injective. We provide a few important theoretical novelties, considering also exponential subclasses of degenerate C-distribution semigroups and degenerate C-(ultra)distribution cosine functions.

The talk is based on collaborations with prof. Stevan Pilipović and prof. Marko Kostić.

[1] A. Favini, A. Yagi, *Degenerate Differential Equations in Banach Spaces,* Chapman and Hall/CRC Pure and Applied Mathematics, New York, 1998.

[2] M. Kostić, *Abstract Degenerate Volterra Integro-Differential Equations: Linear Theory and Applications*, Book Manuscript, 2016.

[3] P. C. Kunstmann, *Distribution semigroups and abstract Cauchy problems*, Trans. Amer. Math. Soc. **351** (1999), 837–856.

[4] I. V. Melnikova, U. A. Anufrieva, V. Yu. Ushkov, *Degenerate distribution semigroups and well-posedness of the Cauchy problem,* Integral Transform Special Functions **6** (1998), 247–256.

Solutions, bi-solutions and Green's operators for non-smooth wave equations

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Classically singularities in general relativity are studied by looking at obstructions to the evolution of test particles along causal geodesics. An alternative is to consider a singularity as an obstruction to the evolution of a test-field. This point of view was called *generalised hyperbolicity* by Clarke and involves regarding certain singularities as interior points in a spacetime with low regularity and then proving local well-posedness of the wave equation in the rough extension. In recent work [1, 2] it has been shown that generalised hyperbolicity applies to spacetimes with generalised conical singularities and where the metric is only Lipschitz.

In this talk we extend this approach to look at the evolution of quantum fields in spacetimes of low regularity. For quantum fields a key role is played by the Feynman propagator – a bi-solution of the differential equation constructed from the advanced and retarded causal Green's operators G^+ and G^-. For smooth globally hyperbolic spacetimes the existence of these follows from the well-posedness of the initial value problem. Surprisingly in the non-smooth case very little is known about the existence of G^+ and G^-. In this talk we will generalise some ideas found in the proof of the L^2-conjecture in general relativity [3] to give partial answers to the question of the existence of the relevant Green's operators required to construct quantum propagators on non-smooth spacetimes.

[1] Y Sanchez Sanchez, J A Vickers *Generalised hyperbolicity in spacetimes with string-like singularities*, Class. Quantum Grav. **33** (2016) 205002.

[2] Y Sanchez Sanchez, J A Vickers *Generalised hyperbolicity in spacetimes with Lipschitz regularity*, J. Math. Phys. **58** 022502 (2017)

[3] S Klainerman, I Rodnianski, J Szeftel *The bounded L^2 curvature conjecture*, Invent. Math. **202** 91-216 (2015)

New developments in the non-linear theory of generalized functions: optimal embeddings of ultradistributions and hyperfunctions

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In this talk we present an overview of various recent developments concerning the possibility to construct optimal embeddings of ultradistributions and (infra-)hyperfunctions into algebras of generalized functions.
Optimality of the embedding here refers to the preservation of the multiplication of ultradifferentiable functions avoiding any "loss of regularity"; in the hyperfunction case this means preserving the multiplication of real analytic functions. The construction of such optimal embeddings was up to now an important question; our main goal is then to present its solution. The hyperfunction and quasianalytic cases are much more difficult to deal with; in particular, their analysis requires to investigate the solvability of the Cousin problem for vector-valued quasianalytic functions.
The talk is based on collaborative works with A. Debrouwere and H. Vernaeve [1–3].

[1] A. Debrouwere, J. Vindas, *Solution to the first Cousin problem for vector-valued quasianalytic functions*, Ann. Mat. Pura Appl., in press, doi:10.1007/s10231-017-0649-0.

[2] A. Debrouwere, H. Vernaeve, J. Vindas, *Optimal embeddings of ultradistributions into differential algebras,* Monatsh. Math., to appear (preprint: arXiv:1601.03512).

[3] A. Debrouwere, H. Vernaeve, J. Vindas, *A non-linear theory of infrahyperfunctions,* preprint: arXiv:1701.06996.

Toroidal pseudodifferential operators in spaces of ultradistributions on $\mathbb{T}^n$

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In this talk we will study a class of symbols and corresponding pseudodifferential operators of finite order on torus $\mathbb{T}^n$ that act continuously on a space of ultradistributions on $\mathbb{T}^n$, of Beurling and Roumieu type, and develop symbolic calculus for these classes.

The talk is based on collaboration with Jasson Vindas.

[1] M. Ruzhansky, V. Turunen, *Pseudo-Differential Operators and Symmetries: Background Analysis and Advanced Topics*, Birkhäuser, Basel, 2010.

[2] M.Cappiello, L. Rodino, *SG-pseudodiferential operators and Gelfand- Shilov spaces*, Rocky Mountain J. Math. **36** ,**4** (2006), 1117-1148.

[3] B.Prangoski, *Pseudodifferential operators of infinite order in spaces of tempered ultradistributions*, J. Pseudo-Differ. Oper. Appl. **4** (2013), 495-549.

Asymptotic expansions of thick distributions

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The theory of thick distributions was initially introduced to deal with the occurrence of a distributional singularity on the boundary of a domain of integration. In this talk I will introduce the asymptotic expansion of thick distributions. I will give a few examples of such expansion. It should be mentioned that the projection of thick expansion onto the usual distribution space is the same as the asymptotic expansion of usual distributions.

The talk is based on collaborations with Professor Ricardo Estrada.

Stochastic evolution equations with Wick-square nonlinearity

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We study stochastic nonlinear evolution equations of the form

$$u_t(t,x,\omega) = \mathbf{A}\, u(t,x,\omega) + u^{\lozenge 2}(t,x,\omega), \quad u(0,x,\omega) = u^0(x,\omega), \tag{41}$$

where $t \in (0,T]$, $\omega \in \Omega$ and $u(t,\cdot,\omega)$ belongs to a certain Banach algebra X. The operator $\mathbf{A}$ is densely defined, generating a C_0-semigroup. The nonlinear part is given in the Wick-square product form $u^{\lozenge 2} = u \lozenge u$, where $\lozenge$ denotes the Wick product. In order to solve (41), we combine the Wiener-Itô chaos expansion method with the deterministic semigroup theory and theory of nonlinear partial differential equations.

The talk is based on collaborations with T. Levajković, S. Pilipović and D. Seleši.

Theory and applications of boundary-domain integral and pseudodifferential operators

Organizers: E: Mikhailov, D. Natroshvili

Analysis of boundary-domain integral equations for variable-coefficients mixed BVPs in 2D

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In this paper, the mixed boundary value problem for the second order "stationary heat transfer" partial differential equation with variable coefficient is considered in 2D. Using an appropriate parametrix (Levi function), this problem is transformed to some direct segregated Boundary-Domain Integral Equations (BDIEs). Although the theory of BDIEs in 3D is well developed, the BDIEs in 2D need a special consideration due to their different equivalence properties. Consequently, we need to set conditions on the domain or on the associated Sobolev spaces to insure the invertibility of corresponding parametrix-based integral layer potentials and hence the unique sovability of BDIEs. The properties of corresponding potential operators are investigated. The equivalence of the original BVP and the obtained BDIEs are analysed and the invertibility of the BDIE operators is proved.

The talk is based on collaborations with T.T. Dufera and S.E. Mikhailov.

Mixed and crack type dynamical problems of the thermopiezoelectricity theory without energy dissipation

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In this paper, we study mixed and crack type boundary value dynamical problems of the linear theory of thermopiezo-electricity for bodies with inner structure. The model under consideration is based on the Green-Naghdi theory of thermo-piezoelectricity without energy dissipation. This theory permits propagation of thermal waves at finite speed. We investigate a mixed boundary value problem for homogeneous isotropic solids with interior cracks. We derive Green's formulae and prove the corresponding uniqueness theorem. Using the Laplace transform, potential method and theory of pseudodifferential equations on a manifolds with boundary we prove existence of solutions and analyze their asymptotic properties. We describe the explicit algorithm for finding the singularity exponents of the thermo-mechanical and electric fields near the crack edges and near the curves where different types of boundary conditions collide.

The talk is based on collaboration with T. Buchukuri and D. Natroshvili.

Analysis of boundary-domain integral equations for variable coefficient Dirichlet BVP in 2D unbounded domain

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In this paper, the Dirichlet boundary value problem for the second order "stationary heat transfer" elliptic partial differential equation with variable coefficient is considered in unbounded (exterior) two dimensional domain. Using an appropriate parametrix (Levi function), this problem is transformed to some direct segregated Boundary-Domain Integral Equations (BDIEs). Although the theory of BDIEs in 3D is well developed, the BDIEs in 2D need a special consideration due to their different equivalence properties. Consequently, we need to formulate the boundary value problem in appropriate weighted Sobolove spaces which are suitable for unbounded domain. We investigate the properties of corresponding parametrix-based integral layer potentials and hence the unique sovability of BDIEs. The equivalence of the original BVP and the obtained BDIEs is analyzed.

The talk is based on collaborations with Sergey E. Mikhailov.

Weights and BVPs for analytic and generalized analytic functions

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The goal of our talk is to discuss the following topics:

(i) to give solutions of the Riemann and Riemann-Hilbert problems in the frame of new, grand function spaces;

(ii) new weight results for Cauchy singular integral operators as a consequence of BVPs solutions.

Hadamard formula for eigenvalues of the Dirichlet problem for elliptic operators on C^1-domains

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The classical Hadamard formula for the first eigenvalue of the Dirichlet-Laplacian in a bounded domain gives its asymptotic representation when we perturb the reference domain by using a normal shift function. This asymptotic formula was derived under quite restrictive smoothness assumptions on the boundary of the reference domain and the shift function. Here we show that it is sufficient to assume only C^1 smoothness of both the reference and perturbed domains. We demonstrate also that this smoothness is optimal. Namely, the result is not true for Lipschits domains and Lipschits perturbations. For $C^{1,\alpha}$-domains we give an optimal estimate of the remainder term in the asymptotics.

The talk is based on collaborations with Johan Thim.

On indirect boundary integral equations methods and applications

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In the classical indirect method, the solution of Dirichlet (Neumann) problem for Laplace equation is sought in the form of double (single) layer potential. We consider the alternative method in which the choice of form for the solution of Dirichlet (Neumann) problem is a single (double) layer potential. In this talk, we present how to obtain these integral representations. The boundary integral method we use requires neither pseudo-differential operators theory nor hypersingular integrals, but it only hinges on the theory of reducible operators and the theory of differential forms. We also show some applications of our results in the theory of conjugate differential forms and the applicability of our method to different partial differential equations.

The talk is based on collaborations with Alberto Cialdea and Vita Leonessa.

Integral potential operators for Brinkman PDE system with nonsmooth coefficients in exterior domain

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A variational approach is used to define the generalised Newtonian and layer potentials for the nonsmooth-coefficient Brinkman system (which can be considered as a modified Stokes system for viscous fluid flow in porous medium) in a Lipschitz exterior domain in $\mathbb{R}^n$, and prove the mapping properties of the associated operators. Some new weighted Sobolev spaces are introduced to correctly describe behaviour of the pressure field at infinity. We show that the Newtonian and layer potentials provided by the variational approach coincide with the well known Brinkman layer potentials in the integral form in the case of constant coefficient Brinkman system

The talk is based on collaborations with MIRELA KOHR (Babeş-Bolyai University, Cluj-Napoca, ROMANIA).

Boundary-domain integral equations for scalar quasilinear elliptic problems

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The investigation is devoted to the application of localized parametrix approach in the study of three-dimensional boundary value problems (BVP) for quasilinear partial differential equations. Nonlinear problems treated in the paper appear in mathematical modelling of many physical, engineering, biological, and life sciences. We consider the Dirichlet, Neumann, Robin, and mixed type boundary value problems in the classical and weak settings for scalar quasilinear divergence form partial differential equations of elliptic type and reduce them to the corresponding *nonlinear localized boundary-domain integral equations systems* (NBDIE). Our main interest here is to analyze the obtained NBDIE. In particular, we show that the localized NBDIE systems are equivalent to the original nonlinear BVPs for a wide class of quasilinear differential equations in appropriate Hölder and Bessel potential spaces. For particular cases, the iteration method of successive approximations for the nonlinear integral equations systems is substantiated on the basis of the fixed-point theorem. For linear BVPs with variable coefficients the similar approach has been successfully developed in [1,2].

The talk is based on collaboration with Sergey Mikhailov (Brunel University London).

[1] O. Chkadua, S.E. Mikhailov, D. Natroshvili, *Analysis of some localized boundary-domain integral equations.* J. Integral Equations Appl., **21**(2009), No. 3, 407-447.

[2] O. Chkadua, S.E. Mikhailov, D. Natroshvili, *Localized boundary-domain singular integral equations based on harmonic parametrix for divergence-form elliptic PDEs with variable matrix coefficients,* Integral Equations and Operator Theory, **76** (2013), 509-547.

Pseudo-differential equations, boundary value problems and conical potentials

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We study Fredholm properties of elliptic pseudo-differential operators (or equations) in Sobolev–Slobodetskii spaces on manifolds with a boundary [1] but in our case the boundary may be non-smooth [2].
Basic principles for studying such equations are the following:

- a local principle or freezing coefficients principle;
- factorizability principle for an elliptic symbol at boundary point;
- a pluralism principle for singular boundary points which implies distinct types of local operators.

There are certain interesting corollaries for a statement of boundary value problems [3].

[1] G. Eskin, *Boundary value problems for elliptic pseudodifferential equations*, AMS, Providence (1981).

[2] V.B. Vasilyev, *Elliptic equations and boundary value problems in non-smooth domains*, In: Pseudo Differential Operators: Analysis, Applications and Computations. Operator Theory: Advances and Applications. **213**. Birkhäuser, Basel (2011). P.105–121.

[3] V.B. Vasilyev, *New constructions in the theory of elliptic boundary value problems*, In: Integral Methods in Science and Engineering. Birkhäuser, Basel, 2015. P. 629–641.

Wavelet theory and its related topics

Organizers: K. Fujita, A. Morimoto

Gabor transformation on the sphere and its related topic

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In the 9th ISAAC Congress, we studied the Gabor transform of analytic functionals on the sphere in general dimension. Then in the 10th ISAAC Congress, we studied the Gabor transformation on the 2-dimensional sphere and its inverse transformation. In this talk, following our previous results, we will consider some applications by means of the characteristics of the Gabor transform of analytic functionals or square integrable functions on the sphere.

[1] K.Fujita, *Gabor transform of analytic functional on the sphere*, Current Trends in Analysis and its Applications, Proceedings of the 9th ISAAC Congress, Kraków 2013, Springer, 451-458 (2015).

[2] K.Fujita, *Gabor Transformation on the Sphere and Its Inverse Transformation*, New Trends in Analysis and Interdisciplinary Applications, Selected Contributions of the 10th ISAAC Congress, Macau 2015, Springer, 573-580 (2017)

A two-dimensional lapped directional wavelet transform with directional lifting

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We introduce a two-dimensional lapped directional discrete wavelet transform. The proposed transform is biorthogonal, and the directional selectivity of the two-dimensional discrete wavelet transform is improved. The lifting scheme extended with a redundant polyphase decomposition on $\mathbb{Z}^2$ is used for constructing two-dimensional biorthogonal directional filters that generate a set of biorthogonal wavelets. As a specific example, we present biorthogonal wavelets based on B-spline functions and discuss its property.

Detection of rotation angles on image separation problem

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We discuss about image separation problem. We observe linear combinations of source images with rotation and translation. From several observed images, we want to estimate parameters such as the number of source images, mixing coefficients, rotation angles, and translation distances, finally to separate source images. In this talk, we propose detection method of rotation angles by wavelet analysis.

The talk is based on collaborations with professors R. Ashino and T. Mandai.

[1] R. Ashino, S. Kataoka, T. Mandai and A. Morimoto, *Blind image source separations by wavelet analysis*, Appl. Anal., **91(4)** (2012), 617–644.

[2] R. Ashino, T. Mandai, A. Morimoto, *Multistage blind source separations by wavelet analysis*, Int. J. Wavelets Multiresolut. Inf. Process., **12(4)** (2014), 25 pages.

Application of complex continuous wavelet analysis to auditory evoked brain responses

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Auditory evoked brain responses obtained at mid brain that relay parts of the auditory system are used to assist human objective audiometry test. As examples, there are Auditory Brainstem Response (ABR) and Auditory Steady-State Response (ASSR). In this study, we discuss about the results that we applied the one-dimensional complex continuous wavelet analysis (CCWA) to ABR and ASSR.

The talk is based on collaborations with professors R. Ashino and A. Morimoto.

[1] R. Galambos et al, *A 40-Hz auditory potential recorded from the human scalp*, Proc. Nati. Acad. Sci. USA, **78(4)**(1981), 2643–2647.

[2] M. S. John, et al., *MASTER: a Window program for recording multiple auditory steady-state responses*, Comput. Methods Programs Biomed., **61** (1998), 125–150.

[3] N. Ikawa, *Automated averaging of auditory evoked response waveforms using wavelet analysis*, International Journal of Wavelets, Multiresolution and Information Processing, **11(04)**(2013), 1360009-, 1–21.

Open session (contributed talks)

Organizers: J. Fransson, J. Toft

Fractional analogues of some inequalities

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The idea of fractional calculus based on a conversation between Leibniz and L'Hospital. By the 19th century, studies of a number of mathematicians lead to a coherent theory of fractional calculus. Although there are many definitions of fractional derivatives, the most known definitions are Riemann-Liouville and Caputo derivatives.
Because inequalities are useful tools in mathematics, authors began to establish fractional analogues of those being published before.
In this presentation, we will show fractional analogues of Jensen and Wirtinger type inequalities.

The talk is based on collaborations with A. Feza Güvenilir.

[1] G.H. Hardy, J.E. Littlewood, G. Plya, *Inequalities*, Cambridge University Press (1988).

[2] R. Khalil, M. Al Horani, A. Yousef, M. Sababheh, *A new definition of fractional derivative*, J. Comput. Appl. Math. **264** (2014), 65-70.

Boundedness result of solutions for a semilinear evolution equations

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We consider the Cauchy problem for a strongly coupled semi-linear heat equations with some kind of nonlinearity in multi-dimensional space $\mathbb{R}^N$. We see under some conditions on the exponents and on the dimension N, that the existence and uniqueness of global solutions for small data and their asymptotic behaviors are obtained. This observation will be applied to the corresponding system of the damped wave equations in low dimensional space.

[1] M. Escobedo, M. A. Herrero, *Boundedness and blow-up for a semilinear reaction-diffusion equation*, J. Diff. Eqs. **89** (1991), 176–202.

[2] H. Fujita, *On the blo*wing up of solutions of the problem for $u_t = \Delta u + u^{1+\alpha}$, J. Fac. Sci. Univ. Tokyo 13 (1966) 109–124.

[3] T. Cazenave, F. Dickstein, F. Weissler; An equation whose Fujita critical exponent is not given by scaling, Nonlinear Anal. **68**, (2008) 862-874.

Tauberian conditions under which convergence follows from its summability by the logarithmic power series method

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Let (s_n) be a sequence of real numbers. If

$$-\frac{1}{\log(1-x)}\sum_{n=0}^{\infty}\frac{s_n}{n+1}x^{n+1}$$

tends to a finite limit ξ as $x \to 1^-$ in $(0,1)$, then we say that (s_n) is summable to ξ by the logarithmic power series method. It is clear that if the ordinary limit $\lim_{n\to\infty} s_n = \xi$ exists, then the limit

$$\lim_{x\to 1^-} f(x) = \xi$$

also exists. However, the converse implication holds true only under some supplementary, so-called Tauberian condition(s). In this work, we find Tauberian conditions of slowly decreasing type under which convergence of a sequence of real numbers follows from its summability by logarithmic power series method.

The talk is based on the collaboration with SEFA ANIL SEZER (Istanbul Medeniyet University).

[1] Kwee, B., *Some Tauberian theorems for the logarithmic method of summability*, Canad. J. Math. **20** (1968), 1324-1331.

[2] Móricz, F., *Theorems relating to statistical harmonic summability and ordinary convergence of slowly decreasing or oscillating sequences*, Analysis (Munich) **24** (2004), 127-145.

[3] Móricz, F., *Necessary and sufficient Tauberian conditions for the logarithmic summability of functions and sequences*, Studia Math. **219** (2013), 109-121.

Dissipative property for a class of non local evolution equations

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In this work we consider the non local evolution problem

$$\begin{cases} \partial_t u = -u + g(\beta K(f\circ u)(x,t) + \beta h),\ x \in \Omega,\ t \in [0,\infty[; \\ u(x,t) = 0,\ x \in \mathbb{R}^N \setminus \Omega,\ t \in [0,\infty[; \\ u(x,0) = u_0(x),\ x \in \mathbb{R}^N, \end{cases}$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$; $g, f : \mathbb{R} \to \mathbb{R}$ satisfying certain growing condition and K is an integral operator with a symmetric kernel. We prove that Cauchy problem above is well posed, the solutions are smooth with respect to initial conditions, and prove the existence of a global attractor. Futhermore, we exhibit a Lyapunov's functional, concluding that the flow generated, by the solutions of it, has a gradient property.

The talk is based on collaborations with S. H. Da Silva, B. E. P. Lucena.

A one dimensional asymptotic model of blood flow through a curved, elastic blood vessel

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We derive a one dimensional model of blood flow through a curved blood vessel having an anisotropic, laminar and elastic wall structure. The blood vessel is assumed to have a circular cross section of varying radius along its length while having a general curvature and torsion for its given centre line. Constitutive relations of linear elasticity and Newton's second law for the the wall and linearized Navier-Stokes equations for the blood flow are coupled with the help of appropriate boundary conditions. Assuming the thickness of the wall of the vessel to be very small compared to the radius, as well as the radius to be small compared to the length of the vessel, we perform dimension reduction in two steps using asymptotic expansions to obtain a one dimensional model for the blood flow.

The talk is based on collaborations with Prof. Vladimir Kozlov and Prof. Sergei Nazarov.

Fractional order integro-differential equations solution by artificial neural networks approach

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Great care must be taken in considering the fact that neural networks moved in the direction of a systematic world such as applied mathematics and engineering sciences. Such certain movement helped shaping fantastic changes in the numerical solution of complicated cases which are overt in natural phenomena. In the present study, a comprehensive optimization mechanism consisting of a reliable three-layered feed-forward neural network is formed to solve a class of fractional order ordinary integro-differential equations. One point should be kept in mind that the supervised back-propagation type learning algorithm which is based on the gradient descent method, is capable of approximating the mentioned problem on an arbitrary interval to any desired degree of accuracy. Besides, some comparative test problems are given to reveal the flexibility and efficiency of the proposed method.

Integer sub decomposition method for point multiplication over curves with j-invariant 0

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In this work, we revisited the elliptic scalar multiplication method which is called the Integer Sub-Decomposition (ISD). This method was proposed in 2013 and considered as a complement to a well-known GLV method which was introduced by Gallant, Lambert and Vanstone (GLV) in the year 2001. The original ISD method deals with trivial endomorphism which only works on integer number field. By extending the ISD method into the complex quadratic field, more solutions can be obtained. Allowing the ISD to work on the complex quadratic field has enabled this method to be applicable on special curves, curves with j-invariant 0. Curves with j-invariant 0 have a unique endomorphism over the complex number field. The fact that the ISD method requires three endomorphisms, the second and third endomorphism are chosen in such a way that they belong to the same field as the first endomorphism. Several properties tailored to curves with j-invariant 0 has been established in this work and more on going properties are still being develop to generate proper and suitably efficient endomorphisms for implementation purposes.

The talk is based on collaborations with Siti Noor Farwina Mohamad Anwar Antony.

Hemi-slant submanifolds as warped products in a nearly Kaehler manifold

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A hemi-slant (also called pseudo-slant) submanifold is endowed with two orthogonal complementary distributions, one is totally real and the other is slant. This talk is devoted to investigate about results to characterize these submanifolds as warped product subamifolds in a nearly Kaehler manifold. The existence of the two types of possible warped products have been discussed.

A k-dimensional system of Langevin Hadamard-type fractional differential inclusions with $2k$ different fractional orders

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We investigate the existence of solutions for a k-dimensional system of Langevin Hadamard-type fractional differential inclusions with $2k$ different fractional orders. We provide an example to illustrate our main result.

The talk is based on collaborations with Sh. Rezapour.

Yosida approximations for semilinear backward stochastic differential equation in infinite dimensions

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Convergence of Yosida approximations for semilinear backward stochastic differential equations in infinite dimensions driven by a cylindrical Brownian motion is investigated. By Yosida approximation the involved, posssibly unbounded, infinitesmal generator, is approximated by a bounded infinitesmal generator.

The talk is based on collaborations with Hani Adibi, University of Tunis El Manar, Tunisia.

Uniform boundary stabilization of the wave equation with a nonlinear delay term in the boundary conditions

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Let Ω be an open bounded domain of $\mathbb{R}^n$ with boundary Γ of class C^2 which consists of two non-empty parts Γ_1 and Γ_2 such that, $\overline{\Gamma_1} \cap \overline{\Gamma_2} = \emptyset$.
In Ω, we consider a wave equation with a nonlinear delay term in the boundary conditions:

$$u_{tt}(x,t) - \Delta u(x,t) = 0 \qquad \text{in } \Omega \times (0;+\infty), \tag{42}$$

$$u(x,0) = u_0(x), u_t(x,0) = u_1(x) \qquad \text{in } \Omega, \tag{43}$$

$$u(x,t) = 0 \qquad \text{on } \Gamma_1 \times (0,+\infty), \tag{44}$$

$$\frac{\partial u}{\partial \nu}(x,t) = -\alpha_1 f(u_t(x,t)) - \alpha_2 g(u_t(x,t-\tau)) \qquad \text{on } \Gamma_2 \times (0,+\infty), \tag{45}$$

$$u_t(x,-t) = f_0(x,-t) \qquad \text{on } \Gamma_2 \times (0,\tau), \tag{46}$$

where α_1 and α_2 are positive constants and u_0, u_1, f_0 are the initial data which belong to an appropriate Hilbert space. System (42)-(46) is subject to the following assumptions:

(H.1) (i) f is a continuous, monotone, incrasing function on $\mathbb{R}$;
(ii) $f(s)s > 0$ for $s \neq 0$;
(iii) $M_2 s^2 \leq f(s)s \leq M_1 s^2$ for $\mid s \mid \geq 1$ for some $M_1, M_2, 0 < M_2 < M_1$;
(H.2) g is Lipschitz, i.e $\mid g(u) - g(v) \mid \leq L \mid u - v \mid$.
(H.3) There exists $x_0 \in \mathbb{R}^n$ such that $m\nu \leq 0$ on Γ_1 where $m = x - x^0$.

Under the assumptions stated above, we establish uniform decay rates for the solutions.

The talk is based on collaborations with Wassila Ghecham.

Tauberian theorems for statistical convergence and logarithmic statistical summability

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In this work, we establish new Tauberian theorems for the statistical convergence via the general logarithmic control modulo of the oscillatory behavior of a real sequence. In addition, we define statistical logarithmic summability method and extend some Tauberian theorems to this newly introduced method.

The talk is based on collaborations with RAHMET SAVAŞ (Istanbul Medeniyet University) and İBRAHIM ÇANAK (Ege University).

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Trigonometric approximation of functions in generalized Lipschitz class with Muckenhoupt weights

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In this paper we generalized the definition of weighted Lipschitz class $Lip(\alpha, p, w)$, where $w\,(x)$ is a Muckenhoupt weight given by [1] to the weighted class $Lip\,(\xi\,(t)\,, p, w)\,$, where $\xi\,(t)$ is a positive non-decreasing function and determine the degree of approximation of $f \in Lip\,(\xi\,(t)\,, p, w)$ through matrix means of its trigonometric Fourier series. Our results generalize the theorems of Guven [1, 2].

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