TIME INCONSISTENT STOCHASTIC CONTROL THEORY

Tomas Björk
Stockholm School of Economics, Sweden

We present a theory for stochastic control problems which, in various ways, are time inconsistent in the sense that they do not admit a Bellman optimality principle. We attach these problems by viewing them within a game theoretic framework, and we look for subgame perfect Nash equilibrium points. For a general controlled Markov process and a fairly general objective functional we derive an extension of the standard Hamilton-Jacobi-Bellman equation, in the form of a system of non-linear equations, for the determination for the equilibrium strategy as well as the equilibrium value function. We also study some concrete examples.

This talk is based on a joint paper with A. Murgoci.

TAIL ESTIMATES FOR STOCHASTIC FIXED POINT EQUATIONS VIA NONLINEAR RENEWAL THEORY

Jeffrey Collamore
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Stochastic fixed point equations arise in several areas of modern mathematics and have been the focus of much study in applied probability, finance, analysis of algorithms, page ranking of personal web search, and actuarial mathematics. A general stochastic fixed point equation has the form $V \stackrel{\mathcal{D}}{=} f(V)$, where f is a random function satisfying certain regularity conditions and is independent of V. Some standard applications include the ruin problem with investments, which satisfies the SFPE $V \stackrel{\mathcal{D}}{=} (AV + B)^+$, and the GARCH(1,1) financial time series model, which satisfies the SFPE $V \stackrel{\mathcal{D}}{=} AV + B$. For such SFPEs, a classical result of Goldie (1991) characterizes the tail behavior of V, stating that

$$\mathbf{P}(V > u) \sim Cu^{-\xi} \quad \text{as} \quad u \to \infty. \tag{1}$$

In this result, the expression for ξ is explicit, but not the expression for C.
The objective of our work is to propose a new approach to these problems based on nonlinear renewal theory and the theory of Harris recurrent Markov chains. In this context, we introduce a novel "dual" change of measure, yielding (1) but with a new expression for C which is explicit and *computable*. We also obtain an explicit upper bound similar to the Lundberg inequality of insurance mathematics. The expressions we obtain for the constant C are closely connected to the so-called backward recursive sequences of Letac (1986).
Finally, we discuss some extensions of our method to importance sampling and a few related problems.

This talk is based on a joint paper with Anand Vidyashankar.

NONCAUSAL VECTOR AR PROCESSES WITH APPLICATION TO FINANCIAL TIME SERIES

Richard A. Davis
Columbia University, New York

Inference procedures for noncausal autoregressive (AR) models have been well studied and applied in a variety of applications from environmental to financial. For such processes, the observations at time t may depend on both past and future shocks in the system. In this paper, we consider extension of the univariate noncausal AR models to the vector AR (VAR) case. The extension presents several interesting challenges since even a first-order VAR can possess both causal and noncausal components. Assuming a non-Gaussian distribution for the noise, we show how to compute an approximation to the likelihood function. Under suitable conditions, it is shown that the maximum likelihood estimator (MLE) of the vector of AR parameters is asymptotically normal. The estimation procedure is illustrated with a simulation study for a VAR(1) process and with two real data examples.

This talk is based on a joint paper with Li Song.

GRADUATION TECHNIQUES REVISITED

Boualem Djehiche
The Royal Institute of Technology (KTH), Sweden

I will review some recent results on strengthened filtering and detection of trends and cyclical components in (finite dimensional) financial time series. I will also mention some extensions of these results to functional nonparametric regression models.

VISCOSITY SOLUTIONS OF SYSTEMS OF PDES WITH INTERCONNECTED OBSTACLES AND MULTI-MODES SWITCHING PROBLEM

Said Hamadène
Université du Maine, LMM, France

This talk is related to existence and uniqueness, in viscosity sense, of a solution for a system of m variational partial differential inequalities with inter-connected obstacles. A particular case of this system is the deterministic version of the Verification Theorem of the Markovian optimal m-states switching problem. The switching cost functions are arbitrary. Among others, this problem is connected with the valuation of a power plant in the energy market. The main tool is the notion of systems of reflected backward stochastic differential equations with oblique reflection.

Keywords: Real options; Backward stochastic differential equations; Snell envelope; Stopping times ; Switching; Viscosity solution of PDEs; Variational inequalities.

CONTINUOUS TIME MEAN VARIANCE OPTIMIZATION UNDER PARTIAL INFORMATION

Mia Hinnerich
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We study the mean-variance portfolio optimization problem in continuous time under partial information. We assume that the investor can observe the stock price but not the instantaneous return. We assume that the instantaneous return follows a process of Ornstein-Uhlenbeck type. We treat this time inconsistent optimization problem by solving the extended Hamilton Jacobi Bellman equation developed by Björk and Murgoci (2009).

This talk is based on a joint paper with Tomas Björk and Agatha Murgoci.

OPTIMIZATION AND (F)BSDE OF QUADRATIC GROWTH

Peter Imkeller
HU Berlin, Germany

A financial market model is considered on which agents (e.g. insurers) are subject to an exogenous financial risk, which they trade by issuing a risk bond. They are able to invest in a market asset correlated with the exogenous risk. We investigate their utility maximization problem, and calculate bond prices using utility indifference. In the case of exponential utility, this hedging concept is interpreted by means of martingale optimality, and solved with BSDE with drivers of quadratic growth in the control variable. For more general utility functions defined on the whole real line we show that if an optimal strategy exists then it is given in terms of the solution $(X; Y; Z)$ of a fully coupled FBSDE. Conversely if the FBSDE admits a solution $(X; Y; Z)$ then an optimal strategy can be obtained. In the complete market case, an assumption on the risk aversion guarantees that the FBSDE admits a solution for any finite time horizon. As a particular example of our approach we recover the BSDE for exponential utility, and are able to treat non-classical utility functions like the sum of exponential ones. For utility functions defined on the half line we also reduce the maximization problem to the solution of FBSDE connected with the ones obtained by Peng (1993). In complete markets, once again we provide conditions for solvability that are applicable to the power, the logarithmic and some non-classical utilities. In our approach we propose an alternative form of the maximum principle for which the Hamiltonian is reflected in a martingale.

This talk is based on a joint paper with U. Horst, Y. Hu, A. R´eveillac, and J. Zhang.

WAVELET BASED OUTLIER CORRECTION FOR POWER CONTROLLED TURNING POINT DETECTION IN SURVEILLANCE SYSTEMS

Yushu Li
Linnaeus University, Sweden

Detection turning points in unimodel has various applications to time series which have cyclic periods. Related techniques are widely explored in the field of statistical surveillance, that is, on-line turning point detection procedures. This paper will first present a power controlled turning point detection method based on the theory of the likelihood ratio test in statistical surveillance. Next we show how outliers will influence the performance of this methodology. Due to the sensitivity of the surveillance system to outliers, we finally present a wavelet multiresolution (MRA) based outlier elimination approach, which can be combined with the on-line turning point detection process and will then alleviate the false alarm problem introduced by the outliers.

Keywords: Unimodel, Turning point, Statistical surveillance, Outlier, Wavelet multi-resolution, Threshold.

ON KALMAN FILTER APPLICATION FOR RISK ESTIMATION OF DERIVATIVES PORTFOLIO

Mikhail Nechaev
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Clearing houses or huge broker-dealers usually apply portfolio approach for the estimation of traders' liabilities and restriction of their risk. It means that they analyse possible changes of portfolio's value within a given time horizon. For clearing houses (in USA as well as in Europe) there exists a standard methodology called SPAN, developed by Chicago Mercantile Exchange, where scenario approach is implemented. Sometimes this approach fails due to the finite number (16) of considered scenarios. It seems that parametric approach provides better quality, but in this case we need a fast algorithm for the Implied Volatility (IV) curve forecast. By now there exists an algorithm which uses a polynomial approximation for the IV curve merged with Kalman Filtering procedure as a forecasting tool. We develop a new model and provide statistical evidence which shows that approximation of IV curve by a special class of functions give much better results. Correspondent analysis is based on the data from US options market.

ASYMPTOTICS FOR AN EPIDEMICS MODEL

Roger Pettersson
Linnaeus University, Sweden

We consider a one-dimensional epidemics model where the fraction of infected individuals are given by a four-parameter one-dimensional stochastic differential equation as in Iacus (2008). The parameters may describe transmission rates, recovery or removal rates, rate of infection from outside, and size of population. We study asymptotic properties depending on the parameter values and also whether we have recovery or removal.

FORWARD-BACKWARD SDE GAMES AND OPTIMAL STRATEGIES UNDER MODEL UNCERTAINTY FOR GENERAL UTILITIES

Agnès Sulem
INRIA Rocquencourt, France

We consider a general stochastic system described by a controlled Itô-Lévy process. The performance functional is expressed as the Q-expectation of an integrated profit rate plus a terminal payoff, where Q is a probability measure absolutely continuous with respect to the original probability measure P of the system. We may regard Q as a *scenario measure* controlled by the market or the environment. If Q is uncertain, the agent might seek the strategy which maximizes the performance in the worst possible choice of Q. This gives a *worst case scenario control* problem, which can be formulated as a zero-sum *stochastic differential game* between the agent and the market. We write the performance functional as the value at time $t = 0$ of the solution of an associated *backward* stochastic differential equation (BSDE). Thus we arrive at a stochastic differential game of a system of *forward-backward* SDEs that we study by a maximum principle approach. This method is then applied to study optimal portfolio and consumption problems under model uncertainty and general utility. Using the solution for linear *Malliavin–differential type equations* we arrive at a set of equations which determine the optimal portfolio and consumption of the agent and the corresponding optimal portfolio scenario measure of the market.

This talk is based on a joint paper with B. Øksendal.

OPTIMAL CONTROL OF SPDES WITH DELAY

Bernt Øksendal
University of Oslo, Norway

We study optimal control problems for (time-) delayed stochastic partial differential equations with jumps. We establish sufficient and necessary (Pontryagin type) maximum principles for an optimal control of such systems. The associated adjoint processes are shown to satisfy a (time-) advanced backward stochastic partial differential equation (ABSPDE). Results on existence and uniqueness of solutions of such ABSPDEs are shown. The results are illustrated by an application to optimal consumption from a financial or a biological system modeled by a stochastic reaction-diffusion equation with delay, and to optimal portfolio in market involving interest rate derivatives with delay.

This talk is based on a joint paper with Agnès Sulem and Tusheng Zhang.

NONLINEAR MARKOV PROCESSES

Wei Yang
University of Warwick, Great Britain

A general problem of optimal control of a nonlinear Markov process (or a mean-field process) is considered. Its solution is expressed via the Cauchy problem for an infinite dimensional Hamiltonian-Jacob-Bellman equation with a viscosity solution. First, we prove the well-posedness of the Cauchy problem using discrete time approximation. Then we prove the convergence of the corresponding controlled interacting particle system to the optimal controlled nonlinear Markov processes. Our results generalise and unify the variety of concrete schemes which are used in practice where the evolutions are usually defined on discrete state spaces.