Optimal public transport policy in medium sized cities
Disa Asplund\textsuperscript{1,2,3} and Roger Pyddoke\textsuperscript{1,2,3}
\textsuperscript{1}The Swedish National Road and Transport Research Institute (VTI)
\textsuperscript{2}Division of Transport Economics
\textsuperscript{2}Centre for Transport Studies, Stockholm
\textsuperscript{3}K2 – The Swedish Knowledge Center for Public Transport
CTS Working Paper 2018:1

Abstract
This paper evaluates welfare effects of optimization of fares and frequencies for bus services in small cities. The model used represents both congestion on streets and crowding in public transport vehicles and is calibrated with data for the Swedish city of Uppsala. Four policies are evaluated: optimal fares with unchanged baseline frequencies, optimal frequencies with unchanged baseline fares, simultaneous optimization of fares and frequencies, and finally a so-called Pareto scenario where frequencies and fares are optimized subject to the condition that no consumer group (defined by zone, time period, and origin-destination pair) should be worse off in terms of generalized cost of each trip. The results indicate that there would be large, robust welfare gains from reducing public transport supply in Uppsala, especially in the outer zone of the city where reductions of supply compared with the current situation are large. The welfare gains from adjusting fares would be smaller. The large reductions in consumer welfare in the welfare optimum, however, are likely to be controversial. In the Pareto scenario, almost all potential social welfare gains from the welfare optimal scenario are achieved with no consumer in any zone or time period being worse off compared with the baseline policy. In this scenario, the total number of public transport passengers is increased, and emissions are reduced compared with the current situation.

Keywords: Public transport, bus, demand model, fares, frequencies, supply, optimization, urban, welfare.

JEL codes: R41, R48, R10
ACKNOWLEDGEMENTS

This study has been financed by K2 – The Swedish Knowledge Center for Public Transport and Centre for Transport Studies, Stockholm. We thank Urbanet Analys AB for generously supplying SAMPERS OD-matrix data aggregated to zones and Stefan Adolfsson and colleagues at Public Transit Department UL for comments. We also wish to thank Maria Börjesson and Karin Brundell-Freij for excellent reviewing and comments, and Stef Proost for inspiring discussions on how to model congestion and public transport in cities.

1 INTRODUCTION

The development of urban passenger transport faces important challenges. The policy objectives are to improve accessibility for citizens and to reduce congestion and emissions with efficient policy instruments. Public bus transport in medium sized cities may play an important role in this context. Another challenge is that continued urbanization can be expected, leading to increased urban populations. Both challenges are perceived to require increased capacity in PT. A third related challenge, for PT in Sweden, is that the operating costs have increased rapidly over the past decade (SKL, 2017), raising questions about the possibility of managing demand for the costliest and most congested times and routes. These questions are addressed through optimization of fares and supply in the so-called BUPOV\(^1\) model, proposed in the present study.

The general tendency in public transport (PT) in Sweden has been for a strong growth in supply and demand (Nilsson 2011 and SKL 2017). From 2005 to 2015 the total number of bus boardings in Sweden grew by 32 percent while the number of supply km grew by 22 percent (Trafikanalys, 2016). In medium sized cities in Sweden supply increased by 20 percent and bus boardings by 23 from 2006 to 2011 (Pydöke and Śwärth 2017), while costs increased by 44 percent. This can be interpreted as a response to the goals presented above. The pursuit of these goals and too little emphasis on reducing externalities from car transport, can risk leading to supply of PT beyond where additional costs are motivated by corresponding benefits not only in Sweden.

From a welfare theoretic perspective there are four important motives for using pricing and supply of public transport as policy instruments. The first is if the perception of local decision makers of the potential for realizing welfare gains from shorter waiting times and less crowding in public transport by increased frequencies is underestimated. The second is if congestion in streets and other externalities may be reduced by subsidizing public transport supply. The third is if the true alternative cost of public funds, either from tax collection or in the use of public funds is under- or over-estimated. The fourth is if policies on a national level to price CO\(_2\)-emissions may be restricted to too low levels.

\(^1\) From Swedish: Bussutbud- och grissättning – optimeringsverktyg (bus supply and pricing – optimization tool).
Politics is frequently reluctant to price externalities when this is perceived to be hurtful strong interest groups. A solution has been to use alternative policy instruments that can reduce externalities without raising the cost for the strong interest group. Subsidizing public transport is such an alternative to pricing congestion.

Subsidizing increased public transport supply reasonably presupposes that the cost for can be justified by benefits from reduced congestion in streets, waiting times and crowding in public transport. Increasing public transport supply without following examining costs and benefits runs the risk of leading to over-supply and over-subsidization of public transport. This study is therefore motivated by a wish to calculate the magnitude of costs and benefits from different levels of supply of public transport for a medium sized Swedish city—Uppsala. The benefits to a large extent depend on the initial levels of supply, costs, prices, mode shares, congestion, crowding and the propensity to change from car transport.

The congestion problems in larger cities have been well studied in the literature. So, has the optimal design of public transport in larger cities which will be shown in the literature review below. The main motivation for this study is that neither the relative magnitude of the respective market and policy failures nor the optimal policies and their potentials are well known for cities smaller than the largest ones.

The aim of this paper is to use the good availability data on origin and destination and demand, supply of bus services in Swedish medium sized cities to explore the socially efficient potential to use public transport pricing and frequencies, both to capture the full benefits of public transport provision and to reduce externalities associated with road congestion and other road use externalities.

The theoretical foundations for the literature on optimal fare differentiation of PT can be found in Vickrey (1963) and Mohring (1972). Vickrey (1963) argued that it may be worthwhile to price peak hour trips higher than off-peak trips to avoid crowding in PT or expensive investments in new capacity for peak use only and to promote higher utilization during off-peak hours. Mohring (1972) found that an increased number of PT users means that the supply can be expanded, leading to higher frequencies and hence higher benefits to all users, and therefore providing a basic rationale for PT subsidies. A further justification for subsidizing PT, when politicians are reluctant to price road congestion, is that subsidization of PT may reduce congestion.

Several papers have developed models to optimize congestion charges and PT fares and frequencies for large cities. In some of these models, PT supply is also optimized, and in others, optimal pricing of the car alternative is considered. Consideration of distributional effects, emissions, and crowding also differs between studies. Insights from these studies include that there may be a potential for differentiation of PT fares and congestion charges between peak and off-peak hours and between different geographical areas.
Dender and Proost (2008) used a numerical model for optimizing public transport fares and car taxes in large cities differentiating for peak and off-peak. The supply was adjusted proportionally and not optimized. The model is applied to data from Brussels and London, and results indicate that optimal taxes and fares in most cases should be higher than current.

Parry and Small (2009) presented a general model for optimal subsidization of public transport with the agency optimizing fares for given occupancy. The model incorporated decreasing average costs for additional passengers, large reduction of car congestion when public transport ridership increases and waiting time reductions. However, also here frequency was adjusted proportionately and no differentiation (in time or space) of fares were made.

Basso et al. (2011) analyzed urban congestion management policies in a general setting, using a simple representative street kilometer approach, allowing users to choose between car, bus or biking. The model included congestion interactions between cars and buses; and allowed for optimization of frequency, vehicle size, spacing between stops and percentage of capacity to be dedicated to bus lanes.

Kilani et al. (2014) performed a case study for Paris to study road and public transport pricing in three of four concentric zones in peak and off-peak, utilizing an existing [extensive type] transport model. The model optimized prices but adjusted PT supply proportionately. The results indicated that peak bus fares should be increased as crowding is important in Paris.

Tirachini et al. (2014) aim at both building an extended model for optimizing social welfare with the following decision variables: congestion tolls and bus supply (e.g. fares, frequency, bus design, bus stop design, ticket collection method) and applying it to a corridor in North Sydney for which good data is available. The model distinguishes between 12 zones is built with emphasis on how the perception of crowding in buses is perceived and interacts with interior bus design. Optimal frequency is shown to be sensitive to crowding costs, indicating the importance of understanding crowding costs better. The paper models only one fare for all trips along the corridor and one toll price.

Basso and Silva (2014) used a similar representative kilometer approach as Basso et al. 2011 for the road network in London and Santiago de Chile and compared three major measures: congestion tolls, a number of design variables of the bus network in peak and off-peak (e.g. fares, frequency of service, distance between bus stops) and dedicated bus lanes. The study evaluated the measures by social welfare maximization and distributional analysis. For London, they found that congestion tolls and dedicated bus lanes were more efficient than subsidized bus trips.

Börjesson et al. (2017) used a similar approach as Basso et al. (2011, 2014) for a corridor in Stockholm, but excluding the bike mode. The results indicated that
there may be a substantial potential to reduce supply in off-peak periods and to differentiate fares between peak and off-peak hours.

In Pyddoke et al. (2017) the transport system in Uppsala, Sweden, is modeled and some policy packages analyzed with respect to environmental effects, financial costs, and changes in mode shares. The results suggest that both increases in frequency and reduced fares lead to substantial cost increases without the corresponding reasonable increases in ridership. However, the study did not estimate the user costs and benefits, which are important components for a welfare analysis. Hence, there is a need to study also this issue, and to perform a welfare calculation.

The purpose of the present study is to develop a new model of urban bus services and alternative travel modes – the BUPOV model – inspired by Börjesson et al. (2017). A further purpose is to calibrate this model and optimize total welfare with respect to the decision variables peak and off-peak fares and frequency of bus services for a small or medium sized city. As for our scope, we capture major welfare effects of all trips that begin or end in Uppsala, yet only the parts of those trips that occur within the city borders. That is, possible non-internalized external effects outside of Uppsala (for example congestion effects in Stockholm) resulting from trips beginning or ending in Uppsala are outside the scope of the present study.

Previous studies have studied large cities with substantial congestion. There is therefore uncertainty about what relevance congestion has for smaller and medium-sized cities. Thus, the present study aims to analyze this question. The contribution is to model optimal bus pricing and frequency in a medium sized city for which detailed data on origin and destination with modal choice and in particular street congestion, bus use and occupation rates are available. This data is important, as the baseline of the model represents real occupancy rates by zone and time period. Like earlier studies, the present model simplifies by representing average conditions (travel, walking and waiting times), crowding in buses etc. The bus frequency is optimized instead of adjusting it proportionally to demand, like in Parry and Small (2009) and Kilani et al. (2014). In contrast to Basso et al. (2011, 2014) and Börjesson et al. (2017), who use a representative kilometer approach, we represent differences in demand and crowding across a representative line, e.g. it is more directly focused on public transport. Hence, a novelty is that a simple method to account for variation in demand over the representative line, e.g. lower occupancy levels in close to each end, and higher occupancy levels at the city center. To this end, we develop a simple, general model for variation of occupancy, which needs only data of mean occupancy for calibration, in contrast to the data intensive approach in Tirachini et al. (2014). A contribution in relation to Tirachini et al. (2014) is that the fares in the present paper are differentiated by both time period and zone. To summarize, the main model contribution is to represent 1 a complete public transport system, accounting for the most important effects of policy reforms, while maintaining a

\[2\] From national travel survey and demand model.

\[3\] From regional public transport authority Upplands lokaltrafik.
high level of simplicity. The empirical contribution is to model a midsized city, using high resolution data.

The main findings in this paper is that large welfare increases can be achieved by reducing frequencies. Lower frequencies reduce both operating costs and road congestion. In addition, lower fares contribute, but considerably less, to increasing welfare. Or put differently, crowding in buses in Uppsala does not currently justify general increases in fares or frequencies. The finding that supply should be reduced is in line with those in Börjesson et al. (2017), while the results regarding the optimal fares are rather different. Börjesson et al. 2017 found – in line with the theories of Vickrey (1963) – that peak hour fares should be increased and off-peak fares reduced. In the present study, the peak pricing result holds, but the general price level is reduced while this was not the case in Börjesson et al. This difference could be due to a low degree of crowding (onboard buses) in Uppsala in combination with substantial congestion (on streets) which is currently not priced.

The remainder of the paper is organized as follows. The model is presented in Section 2. In Section 3, the data used are presented and a description of Uppsala is given. Simulation results are presented in Section 4 and, finally, findings and limitations are discussed in Section 5.

2 METHOD

2.1 Model overview

The model in this paper is intended to represent the effects of fares and frequencies on mode choice, timing of trips and welfare in small and medium sized cities with one PT mode. The model is based on a radial spatial representation of a city with two zones – one inner and one outer. The analysis is restricted to weekday traffic, divided into two time-period categories: peak and off-peak. This representation makes it possible to analyze fares and frequencies that are differentiated in time and space. Since the studied policy measures are evaluated on zone level with trips aggregated, route choices within each zone are assumed to be unaffected, and hence route choice is not modeled. This approach implies a limitation as the rebound effect from reduced congestion in the city center is not fully accounted for, since some traffic going around the city to avoid crowding may switch route to going through the city center. Such changes are not represented. In the empirical section it is shown that simplification may be of limited importance.

BUPOV is based on detailed data on the present travel behavior in terms of OD matrices and implicitly represents the present population density, but does not represent changes population or place of residence. There are two time periods, i.e., peak and off-peak, and we assume that the city has two zones, i.e., the city center (inner zone) and the outer city (outer zone). The travelers can choose between three modes of transportation: car, bus, and walking/bicycle.
The choice of travel alternative depends on monetary costs, road congestion, crowding in PT vehicles, and time gains and losses due to changes in PT frequencies. In addition to the effects of PT policies for the producers and consumers, there are effects for the time cost of freight traffic and the health effects (noise, air pollution), as well as environmental effects primarily in terms of, e.g., carbon dioxide emissions. The changes in the regional public transport authority’s (RPTA) financial results are evaluated with a marginal cost of public funds (MCWF) factor. In an optimum, this should correspond both to the marginal welfare costs of raising one further unit of tax revenue and the marginal valuation of using one further unit of public funds.

We model three types of origin-destination (OD) pairs: within the inner zone (inner), between zones in any direction (inter), and within the outer zone (outer). Each type constitutes a separate (isolated) demand system, but they are linked to each other through a sharing of space, both inside the vehicles and on the streets. The demand for a travel alternative (mode m and time period t, for a trip for an OD pair) is modeled through a change from the demand in a reference situation as:

\[ \Delta D_{m,t,OD} = \Delta D_{m,t,OD}(p, f, o, \delta | \varepsilon), \]  

(1)

where \( p \) is price (fare)\(^5\), \( f \) is frequency, \( o \) is level of occupancy in PT, \( \delta \) is traffic delay (for buses and cars), and \( \varepsilon \) is a matrix of elasticities. The total travel demand for each OD pair is assumed to be constant in terms of number of starts and destinations (destination choice is not assumed to be affected), and route choices with in each mode are not assumed to be affected at an aggregate level by the variables in eq. (1).\(^6\) But the choice of mode and the choice of timing of each trip are flexible. This implies that when the demand decreases for a mode in a time period, these trips are allocated among the other time periods and modes proportionally to the initial demand for each other mode and time period and vice versa for demand increases.

The adjustment to a new user equilibrium caused by a change in a policy variable (such as frequency) is done by successive iterations of demand calculations of consumer travel choices, congestion, and in-vehicle crowding in buses. In the baseline case, demand is assumed to be in a steady state, but if a policy reform with respect to frequency or fare is introduced, a new steady state is approached through iteration. The levels of congestion and crowding affect the generalized cost of each travel alternative, which means that some travelers adjust their travel choice when these levels change, meaning that congestion and crowding will again be updated. This iteration process will continue until the model reaches a new steady state.

---

\(^4\) Not modeling the direction of trips (i.e., towards and away from the city center) in morning versus afternoon peak hours is a simplification that may lead to an underestimation of crowding.

\(^5\) The monetary cost of the car alternative (parking fees and distance based costs) does not change.

\(^6\) The network and routing is not handled in the model, but the model is based on mean travel lengths and travel times for each mode and OD pair from a separate routing model.
Consider the following example: First the frequency of bus transport is increased in a zone in a time period. This increases the attractiveness of and therefore the demand for bus transport in the respective zone and time period. This demand is taken from other travel alternatives for the same OD pair. The adjustment in demand in this case decreases the congestion on streets and but increases crowding on buses. These effects cause secondary demand effects calculated in the second iteration, and so forth.

It is assumed that the walking and bicycle mode does not interact with congestion. That is, walkers and cyclists do not experience congestion and do not contribute to congestion for other modes. The costs associated with walking and cycling are therefore independent of the level of motorized traffic, which obviously is a simplification.

A further simplifying assumption is that in each iteration, travelers can only switch to the travel alternatives closest to the current one. Closest here refers to either a change in departure time or a change in mode, but not to a simultaneous change in both. This means that they cannot change both departure time and mode in the same iteration. However, if crowding or congestion is affected in the new travel alternative, spillover effects are allowed in the next iteration, where travelers are allowed again to adjust their travel behavior, so that in the end all possible travel alternatives are affected.

We do not assume any specific functional form of the utility function of the consumers (travelers) but use own price elasticities for PT and car as a basis for calculating other elasticities. That is, the demand model is based on the difference in generalized cost compared with the baseline case (in order to avoid path dependence, moves are always applied to the baseline case).

It is assumed that frequency is proportional to supply and that waiting time and changing time are inversely proportional to frequency. We assume the simplified model of the city and its bus route network depicted in Figure 1.

---

7 The assumption is that demand for each travel alternative is approximately linear prices. Börjesson et al. (2017) use a quadratic utility function that result in such a linear demand system. However, this formulation implies the following relationship $\frac{\partial D_j}{\partial C_j} = \frac{\partial D_i}{\partial C_i}$, i.e. the absolute demand response due to a fixed absolute change in generalized cost is equal for all travel alternatives, which may seem too restrictive in practice. See Appendix A.
The largest circle represents the edge of the city, which coincides with the two terminuses for each line (T) and the smallest circle the border (B) between the inner and outer zones. The straight lines represent the bus routes. Although Figure 1 gives the impression that changes are only possible in the city center, it is assumed that changing between routes is possible at multiple unspecified points (as in reality, routes are not straight lines). The assumption is that the route network is fixed, and that increases and reductions in supply are made proportionally to the original distribution of supply within each time period and zone. It is assumed that net boarding and alighting occur at a constant positive rate (bus stops are not represented in the model) for buses going toward the city center and at a constant negative rate for buses moving outwards. This implies the model for occupancy per bus route depicted in Figure 2.

Figure 2 Stylized model of occupancy variation for each bus route in baseline scenario

\( d = \) Distance
\( \bar{\alpha} = \) Mean occupancy
\( \bar{\alpha}_o = \) Mean occupancy in outer zone
\( Q_i = \) Inner passenger flow (person-km/one-way bus)
\( Q_o = \) Outer passenger flow (person-km/one-way bus)

This representation of occupancy represents a simplification as there are substantial differences in occupancy between different bus routes in the same period and the same zone in Uppsala. For our purposes, it is assumed that this representation is sufficient as the ambition here is to examine whether there is a
potential for adjusting frequency. In Figure 3, the real variation in occupancy over Line 1 in Uppsala is depicted, as an example comparison with the model version in Figure 2.

![Graph showing variation in occupancy across Line 1 in Uppsala.](image)

Figure 3 Variation in occupancy across Line 1 (in pers./vehicle).

When the number of passengers changes from baseline, the slope in Figure 2 changes and the slopes between T and B are allowed to differ from the one between B and C. If in addition the supply differs between the inner and outer zones, there will be a kink in the occupancy at B. However, we do not account for the extra transfer this would imply, which is not very realistic. The idea is to give a hint to if there’s potential for differentiation (in space) of supply. If so, possible means of doing so in practice includes shortening or prolonging some routes, or introduction of new, shorter routes. Such measures would require additional analysis on a more detailed route level.

Next follows a formal presentation of the central equations. A list of parameters and variables can be found in Appendix B, and a more complete representation of the model can be found in Appendices C and D.

The generalized consumer cost per car trip in each OD pair, time period (TP), and iteration (i) is:

\[
G_{OD, TP, i}^{\text{car}} = \frac{DC_{OD} + p_{OD, TP}^{\text{car}}}{a_{car}} + \sum_{z} \left( V_{OD, z, TP, i}^{\text{int.car}}, \tau_{OD, z, TP, i}^{\text{int.car}} \right),
\]

where DC is the distance cost per car (including capital, fuel and wear and tear), \( p_{OD, TP}^{\text{car}} \) is the mean parking fee paid per car, OD pair and time period.

The generalized consumer cost per PT trip in each zone, time period, and iteration is:
Socially optimal fares and frequencies for urban bus services in small cities

\[ GC_{OD,TP,i}^{PT} = p_{OD,TP}^{PT} + \sum_{j=\text{wait,walk,chg}} (V_{O,TP}^{j,PT} \cdot t_{OD,TP}^{j}) + \sum_{z} (V_{O,TP}^{z,PT} \cdot t_{OD,TP}^{z}) \]

where \( p_{OD,TP}^{PT} \) is the fare per OD pair and time period, \( j \) denotes trip components other than in-vehicle time, \( \text{wait} \) denotes the waiting time, \( \text{walk} \) walking time connecting to the bus stops, and \( \text{chg} \) changing time between bus routes for each trip.

The change in number of trips per mode, OD pair, and time period due to a policy reform is (in iteration \( i \));

\[ \Delta D_{OD,m,TP,i}^{\text{tot}} = \Delta \bar{D}_{OD,m,TP,i} + \sum_{\bar{m},TP} (-\Delta \bar{D}_{OD,\bar{m},TP,i} \cdot \theta_{\bar{m},TP}^{OD,m,TP}) \]

where

\[ \Delta \bar{D}_{OD,m,TP,i} = \Delta GC_{OD,m,TP,i}^{m} \cdot \varepsilon_{m,TP} \]

is the partial change in demand resulting from changes in own generalized cost of each travel alternative \((m, TP)\).

\[ \Delta GC_{OD,m,TP,i}^{m} = GC_{OD,m,TP,i}^{m} - GC_{OD,m,TP,0}^{m} \]

\( \varepsilon_{m,TP} \) is the own generalized cost elasticity (which is derived from the own price elasticity\(^8\)).

\( \theta_{\bar{m},TP}^{OD,m,TP} \) is the share of changes in trips in one alternative \((m, TP)\) that is a result of changes in the generalized cost in another alternative \((\bar{m}, \bar{TP})\). For the closest travel alternatives, this is proportional to travel demand in iteration 0, and for other travel alternatives this parameter is zero. That is, the distribution of moves of trips to the three closest alternatives is proportional\(^9\) to number of trips in each of these three alternatives in the baseline scenario.

As a last step, the total number of trips for each travel alternative within each OD pair is updated as:

\[ D_{OD,m,TP,i+1} = D_{OD,m,TP,i} + \Delta D_{OD,m,TP,i}^{\text{tot}} \]

Eqs. (2–7) are run in a recursive loop (where \( i \) is increased by 1 for each iteration) until the system reaches the user equilibrium; that is, the first iteration when there is no substantial difference between any variable compared with in the previous iteration.

---

\(^8\) \( \varepsilon_{m,TP} = \left. \frac{\partial p_{OD,TP}^{m}}{\partial p_{OD,TP}^{m}} \right|_{\text{cost}} \cdot \frac{\Sigma_{m,TP} (GC_{OD,m,TP}^{m} / p_{OD,TP}^{m,OD,m,TP})}{\Sigma_{OD} (p_{OD,TP}^{m,OD,m,TP})} \)

\(^9\) The proportional assumption could be categorized as a naive assumption, due to lack of data. It may be reasonable when comparing modes during the same time period, but may be problematic when comparing mode choice with timing choice. The model can easily be updated in this regard as new data on the subject becomes available.
The change in consumer surplus (due to a policy change) compared with baseline is defined by the rule of one-half, for each mode\textsuperscript{10}, time period and OD pair (in iteration \(i\)), as:

\[
\Delta C_{S_{i}} = - \sum_{m,O D,T P} \left[ \Delta G C_{O D,T P,i}^m \cdot \frac{D_{O D,m,T P,i}^T + D_{O D,m,T P,0}}{2} \right].
\]

(8)

The total cost of providing supply in each zone and time period is:

\[
c_{z,T P,i} = (c_{d} \cdot d_{z}^2 + (c_{t} + k_{T P,i} \cdot c_{k}) \cdot (1 + \delta_{z,T P,i} \cdot t_{i}) \cdot \theta_{z} \cdot S_{z,T P} \cdot T_{T P},
\]

(9)

where, \(c_{d}\) is cost per distance for bus (fuel and wear and tear), \(c_{t}\) is the running cost per hour for bus (wage of drivers), \(c_{k}\) is the capital cost per hour for bus, \(\delta_{z,T P,i}\) is delay due to congestion, \(t_{i}\) is the mean time of each one-way trip, \(\theta_{z}\) is the share of the route that is in zone \(z\), \(S_{z,T P}\) is the bus supply in departures per hour, and \(T_{T P}\) is the length of time period. \(k_{T P,i}\) is a dummy type variable\textsuperscript{11}, indicating if capital cost should be bourn by the time period in question (typically 1 for peak and typically 0 for OP).

The total welfare effects of a given policy change is:

\[
\Delta W_{i} = \Delta C_{S_{i}} + (1 + \tau) \cdot (\Delta P_{S_{i}} + \Delta P_{R_{i}}) + \Delta C_{T_{i}} + \Delta E_{i},
\]

(10)

where \(\tau\) is the MCPF, \(\Delta P_{S_{i}}\) denote changes in producer surplus, \(\Delta P_{R_{i}}\) denotes the total net benefit from changes in parking revenues, \(\Delta C_{T_{i}}\) denotes congestion benefits for trucks, and \(\Delta E_{i}\) denotes the net social cost of other external effects, all compared with baseline. It is assumed that all parking lots are publicly own (simplification) and that parking is marginally cost based;\textsuperscript{12} that is, the total net benefit from changes in parking revenues is \(\Delta P_{R_{i}} = 0\).

The welfare optima given different restrictions are defined as:

\[
\max_{S,f}(\Delta W_{i,*}(\xi, \Psi)),
\]

(11)

where \(S\) and \(p\) are two-dimensional matrices, \(\xi, \Psi\) are a set of restrictions, and \(i *\) denotes the user equilibrium\textsuperscript{13}. \(S\) has the dimensions time period and zone, while \(f\) has the dimensions time period and OD pair.

\textsuperscript{10} However, there is no change in generalized costs for walking/bicycle, so in practice this calculation is performed for car and bus only.

\textsuperscript{11} It is not formally a dummy, since in the unexpected event of exactly equal supply in peak and of peak, it takes the value 0.5 for both time periods.

\textsuperscript{12} In the sensitivity analysis, the alternative assumption that marginal cost is only half of the marginal revenue has been tested; that is that \(\Delta P_{R} = 0.5 \cdot \sum_{O D,T P} p_{O D,T P} \cdot \left( \frac{\Delta R_{O D,c a r,T P}}{\delta_{c a r}} \right) \).

\textsuperscript{13} Note that eq. (37) implies that the policy maker is a Stackelberg leader, setting policy to in anticipation of what the total response (in the last iteration only) will be. That is policy is set one time only and not in every iteration \(i\).
Socially optimal fares and frequencies for urban bus services in small cities

\[
\xi \in \begin{cases} 
  p_{P_{0,TP}}^T \geq p_{P_{I-0,TP}}^T \\
  p_{P_{I-0,TP}}^T \geq p_{P_{0-0,TP}}^T 
\end{cases}
\text{ for each time period, TP}^*.
\]

Thus, within each time period, it should not be cheaper to travel in both zones than in just one.

\[
\Psi \in \emptyset \quad \text{defines the welfare optimum.}
\]

\[
\Psi \in \{ S = S^0 \} \quad \text{defines the optimum given fixed supply.}
\]

\[
\Psi \in \{ p = p^0 \} \quad \text{defines the optimum given fixed fares.}
\]

\[
\Psi \in \{ \Delta G_{OD,TP,i}^m \leq 0 \} \quad \text{for all OD, m, TP, defines the optimum given the restriction of Pareto improvements from the baseline (called Pareto scenario).}
\]

2.2 Concavity properties of the welfare function

With the baseline assumptions of a MCPF of 30 percent and marginal cost-based pricing of parking, the welfare function is concave with respect to supply, but only slightly concave (rather flat) with respect to fares at an aggregate level. However, for each set of supply and fare levels, the welfare function is locally concave with respect to the mix of fares. This means that there are multiple local maximums of the welfare function for each set of restrictions, where some of these are very different with respect to fare level but still close to the global maximum with respect to aggregate welfare. This makes the search for the global maximum complicated. At the same time, the fact that the welfare function is flat reduces the relevance of the global maximum from an economic perspective, and hence there may be more than one possible candidate for an optimal policy. Nevertheless, in the present study we searched for the combination of fares and frequencies that gives a global optimum for each set of restrictions, using a combination of software (non-linear GRG) and ad-hoc manual search methods.\textsuperscript{14}

We cannot be certain that the identified candidates for optima are the true global optima, but we know that we have covered areas surrounding the candidates for optima in the search.

We have also performed sensitivity analyses of the welfare optimum. For empirical uncertainty of parameters, the above described properties of the welfare function hold. However, when the MCPF is removed altogether, the welfare function decreases with fares (at an aggregate level) in the positive region. Also, when parking is overpriced (with respect to marginal cost of parking), then the welfare function loses its concavity properties with respect to supply to some extent.

\textsuperscript{14} We have used a manual algorithm in the search for the global optimum for each set of restrictions, trying different starting points, which are at least testing all combinations of supply = 0.4 or 1 and fares = 0, 0.4, or 1. We have also tested to start with optima for different sets of restrictions, and combining these in various ways. For our best candidate for a global optimum, we performed an expanded search where policy parameters were optimized in smaller groups. That is given the original candidate optimum, only optimizing supply in the inner or outer zone, or optimizing fare for peak or OP in solitary. Repeating this procedure a few times for each group of parameters resulted in a small trimming of each optimum.
3 DATA

Uppsala lies 70 kilometers north of Stockholm (the capital of Sweden) and has one of Sweden’s oldest universities. In 2010, it had 155,000 inhabitants and the urban area covered 51 square kilometers. The urban area was serviced by a network of city buses with 22 routes covering the city.15

Table 1: The urban bus services in Uppsala in 2010. 1 SEK ≈ 0.1 EUR.

<table>
<thead>
<tr>
<th>Total number of routes</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total route net length</td>
<td>430 km</td>
</tr>
<tr>
<td>Trips (boarding passengers)</td>
<td>15.7 million</td>
</tr>
<tr>
<td>Total cost</td>
<td>336 million SEK</td>
</tr>
<tr>
<td>Fare revenue</td>
<td>176 million SEK</td>
</tr>
</tbody>
</table>

Source: Public Transit Department UL (RPTA).

According to a travel survey from 2010 the mode share (per trip) for PT was 11 percent for trips within Uppsala (Uppsala municipality, 2016). Compared with several smaller cities, this is quite high (Pyddoke and Swärdh, 2017), but compared with Sweden’s larger cities (Norheim et al., 2017b), it is low.

The BUPOV model is calibrated with travel data from the national travel surveys and from the Swedish national passenger demand model, and with boarding data from the RPTA. In addition, a more accurate representation of parking fees is applied, based on the present local regulations.

One difference compared with the travel survey is that the present study concerns weekdays only. Another is that we in addition to trips within the city also include trips with an origin or destination outside of Uppsala (see Calibration section in Appendix F). In this study, the distribution of trips across modes, OD pairs, and time periods is taken from the Swedish national travel demand model, SAMPERS.16 This model is regularly updated for the national infrastructure planning. We have used data from a baseline for 2010.17 Because the absolute numbers in the SAMPERS data do not coincide with those from the municipality’s travel survey for 2015 (Uppsala Municipality, 2016) and boarding data from 2014 (UL, 2015), the SAMPERS demand predictions have been scaled to fit with boarding data. For number of trips, see baseline column in Table G1 in Appendix G. Table E1 in Appendix E reports other data from SAMPERS that have been used.

Table 2 reports the mode shares for Uppsala.

---

15 A large network reform reducing the number of routes to 14 was enacted August 14, 2017.
16 The use of SAMPERS data, the aggregation of data, and the representation of congestion and crowding in PT were inspired by the HUT model used in Pyddoke et al. (2017).
17 Our data set was provided by Urbanet Analys AB.
Table 2: Mode shares in Uppsala, percent.

<table>
<thead>
<tr>
<th>Mode</th>
<th>RVU 2010</th>
<th>RVU 2015</th>
<th>Present study*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car</td>
<td>42</td>
<td>37</td>
<td>39</td>
</tr>
<tr>
<td>Bus</td>
<td>12</td>
<td>13</td>
<td>15</td>
</tr>
<tr>
<td>Bicycle / Walk</td>
<td>44</td>
<td>47</td>
<td>47</td>
</tr>
<tr>
<td>Other</td>
<td>3</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: Uppsala Municipality 2015 (Travel Survey Uppsala)
* Refers to a weekday average (differs between peak and off-peak), including trips with origin or destination outside of Uppsala

The distribution of producer costs between staff, capital, and wear and tear is taken from Eliasson (2015). These costs are then scaled up (by about 1.2) to coincide with actual total costs for PT in Uppsala (from the official accounting by Public Transit Department UL, RPTA); see Table 3.

Table 3: Costs per bus.

<table>
<thead>
<tr>
<th>Eliasson (2016)</th>
<th>SEK/year</th>
<th>SEK/h</th>
<th>SEK/km</th>
</tr>
</thead>
<tbody>
<tr>
<td>426,177</td>
<td>285</td>
<td>7.7</td>
<td></td>
</tr>
<tr>
<td>Present study</td>
<td>531,363</td>
<td>355</td>
<td>9.5</td>
</tr>
</tbody>
</table>

BUPOV uses (own) generalized cost elasticities, which are calibrated to match empirical own price elasticities; see Table 4.18

Table 4: Elasticities of demand with respect to (own) price in present study.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Price elasticity from</th>
<th>Time period</th>
<th>Price elasticity</th>
<th>Price/GC in baseline</th>
<th>Resulting GC elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car</td>
<td>Börjesson et al. (2017)</td>
<td>Peak</td>
<td>-0.54</td>
<td>66%</td>
<td>-0.82</td>
</tr>
<tr>
<td></td>
<td>OP</td>
<td></td>
<td>-0.85</td>
<td>60%</td>
<td>-1.42</td>
</tr>
<tr>
<td>PT</td>
<td>Balcombe (2004)</td>
<td>Peak</td>
<td>-0.26</td>
<td>25%</td>
<td>-1.02</td>
</tr>
<tr>
<td></td>
<td>OP</td>
<td></td>
<td>-0.48</td>
<td>25%</td>
<td>-1.89</td>
</tr>
</tbody>
</table>

Street congestion data are from Tomtom (www.tomtom.com/en_gb/trafficindex/city/UPP, January 30, 2017). Costs of crowding and congestion, and the marginal cost of public funds are taken from the Swedish national guidelines on welfare economics of infrastructure investments, ASEK 6 (Swedish Transport Administration, 2016). The marginal external effects of traffic safety, emissions, and noise from cars and buses (including internalization) are calculated for Uppsala based on a combination of ASEK 6, Nilsson and Johansson (2014), Swedish Transport Administration (2015), and ASEK 3 (SIKA, 2005).19 According to these calculations, car trips in

18 Börjesson et al. (2017) used undifferentiated PT elasticities of -0.4.
19 In Samkost (Nilsson and Johansson, 2014), total externality per vehicle-km was 0.22 SEK for cars and 1.64 SEK for heavy vehicles (e.g., buses) on average in Sweden. However, they used a somewhat lower CO2 value than the official one (ASEK 6). Due to the fact that this figure is both hard to
Socially optimal fares and frequencies for urban bus services in small cities

Uppsala have internalization rates (for all calculable externalities except congestion) of slightly more than 100 percent, while emissions from buses are only internalized by about 50 percent (see Table 5). This means that there will be a small welfare gain from an increased number of car trips, not counting the congestion externalities, since the extra tax collected is worth more than the costs of all other externalities including the emission caused, and vice versa for the buses.

Table 5: Internalization rate of emissions

<table>
<thead>
<tr>
<th>Car</th>
<th>Bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner</td>
<td>104%</td>
</tr>
<tr>
<td>Outer</td>
<td>119%</td>
</tr>
</tbody>
</table>

Additional data is reported in Appendix E.

4 RESULTS

In this section, four different policy scenarios are analyzed (see eq. 37 in Section 2.2). In addition, marginal results are available in Appendix G for the interested reader. In the first scenario, *fares only*, fares for each zone and time period are chosen to optimize welfare. In the second, *supply only*, frequencies are chosen for each time period and origin-destination pair to optimize welfare. In the third, *welfare optimum*, fares and frequencies are chosen simultaneously to optimize welfare (without restrictions). Finally, in the *Pareto scenario*, a further restriction is added: that no consumer group (mode, period, or zone) is allowed to have its generalized cost per trip increased. Table 13 shows the resulting levels of the policy variables in optima.

Table 13: Policy parameter values in optima\(^2\) with different restrictions (as percent of baseline).

<table>
<thead>
<tr>
<th>Zone/OD, Time period</th>
<th>Fares only</th>
<th>Supply only</th>
<th>Welfare optimum</th>
<th>Pareto scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply level</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inner, Peak</td>
<td>100%</td>
<td>57%</td>
<td>57%</td>
<td>62%</td>
</tr>
<tr>
<td>Inner, OP</td>
<td>100%</td>
<td>77%</td>
<td>79%</td>
<td>78%</td>
</tr>
<tr>
<td>Outer, Peak</td>
<td>100%</td>
<td>50%</td>
<td>48%</td>
<td>54%</td>
</tr>
<tr>
<td>Outer, OP</td>
<td>100%</td>
<td>53%</td>
<td>55%</td>
<td>57%</td>
</tr>
<tr>
<td>Fare level</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I-I, Peak</td>
<td>30%</td>
<td>100%</td>
<td>69%</td>
<td>53%</td>
</tr>
<tr>
<td>I-I, OP</td>
<td>44%</td>
<td>100%</td>
<td>43%</td>
<td>42%</td>
</tr>
<tr>
<td>I-O, Peak</td>
<td>75%</td>
<td>100%</td>
<td>97%</td>
<td>53%</td>
</tr>
<tr>
<td>I-O, OP</td>
<td>58%</td>
<td>100%</td>
<td>53%</td>
<td>43%</td>
</tr>
</tbody>
</table>

\(^2\) Best candidate found for global optimum for each set of restrictions.
Socially optimal fares and frequencies for urban bus services in small cities

<table>
<thead>
<tr>
<th></th>
<th>0-0, Peak</th>
<th>0-0, OP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>75%</td>
<td>100%</td>
</tr>
<tr>
<td></td>
<td>58%</td>
<td>100%</td>
</tr>
<tr>
<td></td>
<td>97%</td>
<td>53%</td>
</tr>
<tr>
<td></td>
<td>23%</td>
<td>17%</td>
</tr>
</tbody>
</table>

The estimates in Table 13 indicate that the optimal policy involves reductions in both fares (with two exceptions) and supply, and that for some policy parameters, namely supply in the outer zone and most of the fares, the reductions are drastic. The optimal reductions in supply vary from 22 percent (peak in the inner city in the Pareto scenario) to 52 percent (off-peak hours in the outer zone in the supply only and welfare optimum scenarios), and are generally larger in the outer zone than in the inner zone. The table also shows that optimal supply differs less between scenarios than do optimal fares, and that optimal fares depend more on supply than the other way around. That is the welfare function for each set of restriction is rather flat with respect to fare level. Since there is a considerable degree of concavity with respect to supply, this means that there are multiple local optima, where supply is very similar across the local optima, but fares differ a great deal (the pattern is that peak fare is lower than off-peak fare). The "no restrictions" scenario – which results in the welfare optimum – has an inferior local optimum close to the optimum in the Pareto scenario, and the other way around. Some of these inferior local optima are rather close to the global optima for each scenario with respect to the level of aggregate welfare, and hence the Pareto restrictions are able to switch the orders of them.

Table 14 shows the effects in terms of aggregate demand for each mode.

Table 14: Changes in number of trips in the examined optima compared to baseline (in percent)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Base</th>
<th>Policy scenario (optima)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Fares only</td>
<td>Supply only</td>
<td>Welfare optimum</td>
</tr>
<tr>
<td>Car</td>
<td>133,881</td>
<td>-2%</td>
<td>3%</td>
<td>1%</td>
</tr>
<tr>
<td>PT</td>
<td>50,920</td>
<td>14%</td>
<td>-17%</td>
<td>-3%</td>
</tr>
<tr>
<td>WB</td>
<td>161,372</td>
<td>-3%</td>
<td>3%</td>
<td>0%</td>
</tr>
</tbody>
</table>

The fares only and supply only scenarios lead to drastic changes in PT demand, while in the welfare optimum the aggregate effect is rather small (and in the Pareto optimum it is in-between). However, these aggregate demand changes hide larger changes for the various OD pairs and time periods. The relative effects on car trips and WB are rather small, simply because car trips exceed PT trips in total numbers.

Table 15 shows the effects of the policy scenarios (Table 13) and the respective changes in travel alternatives (summarized in Table 14) on congestion (delay) and crowding (occupancy).

21 In the welfare optimal scenario, public transport demand increases substantially for inter trips in the peak hour due to a larger reduction in fares than in frequency. However, for off peak, the effect of lower frequency dominates the effect of lower fares and hence demand decreases. The net result is a small reduction in demand for bus transport in total. In the Pareto scenario, PT demand is increased for all OD pairs in both time periods, except for O-O trips during the OP.
Table 15: Delays and point occupancy in buses in baseline and in the examined optima respectively.

<table>
<thead>
<tr>
<th>Zone</th>
<th>Time period</th>
<th>Mode</th>
<th>Baseline</th>
<th>Policy scenario (optima)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Fares only</td>
</tr>
<tr>
<td>Delay</td>
<td></td>
<td></td>
<td></td>
<td>%</td>
</tr>
<tr>
<td>Inner</td>
<td>Peak</td>
<td>Car+PT</td>
<td>96%</td>
<td>93%</td>
</tr>
<tr>
<td></td>
<td>OP</td>
<td>Car+PT</td>
<td>42%</td>
<td>40%</td>
</tr>
<tr>
<td>Point occupancy (pers./vehicle)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inner</td>
<td>Peak</td>
<td>PT</td>
<td>62%</td>
<td>66%</td>
</tr>
<tr>
<td></td>
<td>OP</td>
<td>PT</td>
<td>48%</td>
<td>56%</td>
</tr>
<tr>
<td>Outer</td>
<td>Peak</td>
<td>PT</td>
<td>27%</td>
<td>28%</td>
</tr>
<tr>
<td></td>
<td>OP</td>
<td>PT</td>
<td>21%</td>
<td>25%</td>
</tr>
</tbody>
</table>

Table 15 shows that congestion is modestly affected, while there are large effects on bus occupancy rates in the three scenarios where supply is reduced. The occupancy increases the most for the peak hours: from around 60 percent to around 100 percent in the inner zone and from around 30 percent to around 50 percent in the outer zone. Table 16 shows the effects on financial outcomes for the RPTA.

Table 16: Financial outcomes for RPTA in SEK per weekday and changes in subsidy level and capacity use of different policy scenarios. Figures are expressed as changes compared with baseline.

<table>
<thead>
<tr>
<th>Financial outcome</th>
<th>Baseline</th>
<th>Fares only</th>
<th>Supply only</th>
<th>Welfare optimum</th>
<th>Pareto scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fare revenue</td>
<td>572,515</td>
<td>-181,588</td>
<td>-95,471</td>
<td>-207,760</td>
<td>-314,829</td>
</tr>
<tr>
<td>Producer costs</td>
<td>1,232,885</td>
<td>-5,852</td>
<td>-538,877</td>
<td>-533,421</td>
<td>-498,235</td>
</tr>
<tr>
<td>Producer surplus,</td>
<td>-660,370</td>
<td>-175,737</td>
<td>+443,406</td>
<td>+325,662</td>
<td>+183,406</td>
</tr>
<tr>
<td>without MCPF</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capacity use in OP</td>
<td>67%</td>
<td>0%</td>
<td>+10%</td>
<td>+14%</td>
<td>+8%</td>
</tr>
</tbody>
</table>

Optimizing welfare with lower fares imposes large costs to the RPTA in terms of lost revenue, which is counteracted by somewhat decreased costs due to shorter rotation times of vehicles due to reduced congestion. There is, however, an interesting potential to optimize supply. The results indicate that doing so may reduce costs by almost 50 percent, suggesting a substantial current excess supply.

In the scenarios where both fares and supply are chosen to optimize welfare, these savings are reduced a bit. The capacity use in OP is substantially improved in the three scenarios where supply is optimized. Next, Table 17 presents the implications in terms of total welfare.

Table 17: Welfare social costs (negative sign) and benefits (positive sign) in SEK per weekday of different policy scenarios. Figures are expressed as changes compared with baseline.
Socially optimal fares and frequencies for urban bus services in small cities

<table>
<thead>
<tr>
<th>Welfare effect</th>
<th>Fares only</th>
<th>Supply only</th>
<th>Welfare optimum</th>
<th>Pareto optimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer surplus</td>
<td>+256,547</td>
<td>-264,291</td>
<td>-81,388</td>
<td>+89,317</td>
</tr>
<tr>
<td>Of which congestion benefits</td>
<td>+21,074</td>
<td>+8,051</td>
<td>+24,049</td>
<td>+35,180</td>
</tr>
<tr>
<td>Producer surplus, without MCPF</td>
<td>-175,737</td>
<td>+443,406</td>
<td>+325,662</td>
<td>+183,406</td>
</tr>
<tr>
<td>Other external benefits*</td>
<td>+477</td>
<td>+1,713</td>
<td>+2,052</td>
<td>+2,108</td>
</tr>
<tr>
<td>Net social benefits (SEK), including MCPF</td>
<td>+28,567</td>
<td>+313,849</td>
<td>+344,024</td>
<td>+329,852</td>
</tr>
</tbody>
</table>

* Net benefits of external effects and taxation of external effects, including congestion benefits for trucks.

Three observations stand out. The first is that substantial improvements in social welfare can be gained by reducing supply in the supply only scenario. The second is that optimization of the fares only scenario does not achieve much compared with the supply only scenario in terms of net welfare. And third, in the Pareto scenario, social welfare can be improved without reducing generalized costs for any identified group (travelers with any mode, for any OD pair, in any time period) in the model. This comes at a 14,000 SEK/weekday loss in social welfare compared with the welfare optimal scenario, which is a small number compared with the producer cost of 1,200,000 SEK/weekday in the baseline scenario.

A sensitivity analysis has also been performed with respect to five key parameters or assumptions. Since the welfare optimum compared with the baseline scenario implies transfers of benefits from consumers to the producer (RPTA), we performed a sensitivity analysis with the extreme value of $MCPF = 0$. The assumption of marginal cost-based pricing of parking has been tested by assuming that marginal costs are only half of the marginal revenues. We also examined the effects of non-differentiated own prices elasticities for PT, using the estimate of -0.4 from Börjesson et al. (2017). A sensitivity analysis of the mean occupancy in PT was performed by increasing the value from 8.5 to 10. The importance of congestion was assessed by removing all street congestion from the analysis. Finally, the value of frequency was examined by increasing it by 50 percent. These six sensitivity analyses can be roughly characterized as basic assumptions (the first two) or empirical uncertainty (the latter four). The results are presented in Table 18 in the form of policy parameters in the welfare optimum.

Table 18: Sensitivity analysis of optimal policy. Policy parameter values in welfare optimum with different sensitivity tests. Baseline optimum corresponds to Welfare optimum in Table 13.
Socially optimal fares and frequencies for urban bus services in small cities

<table>
<thead>
<tr>
<th>Fare level</th>
<th>Inner, Peak</th>
<th>69%</th>
<th>0%</th>
<th>72%</th>
<th>73%</th>
<th>72%</th>
<th>75%</th>
<th>77%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inner, OP</td>
<td>43%</td>
<td>0%</td>
<td>72%</td>
<td>58%</td>
<td>44%</td>
<td>54%</td>
<td>45%</td>
</tr>
<tr>
<td></td>
<td>Inter, Peak</td>
<td>97%</td>
<td>0%</td>
<td>160%</td>
<td>73%</td>
<td>86%</td>
<td>112%</td>
<td>77%</td>
</tr>
<tr>
<td></td>
<td>Inter, OP</td>
<td>53%</td>
<td>0%</td>
<td>103%</td>
<td>58%</td>
<td>52%</td>
<td>69%</td>
<td>52%</td>
</tr>
<tr>
<td>0-O, Peak</td>
<td>97%</td>
<td>0%</td>
<td>53%</td>
<td>73%</td>
<td>86%</td>
<td>112%</td>
<td>77%</td>
<td></td>
</tr>
<tr>
<td>0-O, OP</td>
<td>53%</td>
<td>0%</td>
<td>23%</td>
<td>58%</td>
<td>52%</td>
<td>69%</td>
<td>52%</td>
<td></td>
</tr>
</tbody>
</table>

The directions of changes in Table 18 are as expected. Supply recommendations seem rather robust and supply reductions are smaller in the inner zone than in the outer zone (smallest reduction for inner zone in OP) for all tests. Optimal fare levels are non-robust, but peak fares are higher than OP fares for all tests (they are equal only when all fares are 0). The MCPF = 0 results stand out, in that they prescribe free PT. However, even for this extreme value of MCPF, the model prescribes a reduction of supply, although not as large as in the baseline scenario. For welfare results and mode shares for the sensitivity analyses, see Appendix II. A summary of Appendix II is that net social benefits of is rather robust across sensitivity analyses. PT demand changes the most (about 20%) in the basic assumptions tests, while is rather robust for empirical uncertainty.

5 DISCUSSION

The main aim of this study was to analyze fares and frequencies differentiated in time and space in Uppsala – a medium sized Swedish city – and to optimize them in different scenarios. The most important finding is that optimization of supply indicates a current large over-supply and low rate of capacity utilization of bus service in Uppsala. If the bus occupancy in other medium sized cities is similar to Uppsala’s similar results are likely to be found. Otherwise the generalizability to other cities may be limited. According to SKL (Diagram 5, 2014), Uppsala County’s mean occupancy of 8.5 passengers/vehicle is in the lower half of the distribution among counties in Sweden, with some counties below and the mean is 11 passengers/vehicle. On the other hand, the result of reduced supply is in line with Börjesson et al. (2017), who make a similar analysis for a corridor in Stockholm, which is the largest city in Sweden and has the largest PT share. Stockholm County has the second highest mean occupancy level in Sweden (according to Diagram 5 in SKL, 2014).

The reduction of services in Uppsala has a larger welfare-increasing potential than does optimization of fares. These results support the notion that the pursuit of increased patronage for public transport risks leading to supply beyond where tangible benefits outweigh its social costs. This in itself, is also an important indication that over supply may exist, or even be frequent also in other medium sized cities. These results therefore suggests that RPTA:s should be careful when dimensioning their supply. The simplicity of the BUPOV model, makes it possible perform analogous analyses without extensive costs.
There are however two important caveats to the normative conclusion that supply should be reduced in Uppsala. A first is that the alternative cost for obtaining or using tax revenue may be both larger or smaller. In the central scenario we have used a marginal cost of public funds value of 1.3 whereas we have done a sensitivity analysis with a value 1.0. In the latter case, the optimal frequency is higher and the prices lower than in the baseline optimum. A second, is that we do not have extensive detailed data on occupancy in buses. We can therefore not exclude a high degree of variance in occupancy motivating some degree of hedging against those instances when many passengers want to board.

When the additional marginal cost of public funds is around 30 percent, the total welfare is rather insensitive to fare changes, while these changes have large distributional effects, both among consumers and between the county (financing the RPTA) and the consumers as a group. The results show that the optimal level of supply is only marginally affected by the different model assumptions and has large effects on total net welfare, while welfare-optimal levels of fares are both sensitive to changes in model assumptions and of less importance (from an aggregate welfare point of view). Therefore, a further policy recommendation could be to base supply decisions on welfare calculations, while leaving decisions about fares to elect politicians. In the case of Uppsala, the results indicate that a reduced supply would increase welfare. Optimization in this respect would lead to substantial increases in occupancy rates, especially during peak hours, when it almost doubles.

The results are also in line with Börjesson et al. (2017) in that an optimal policy involves peak pricing (Vickrey, 1963), in that it is optimal to subsidize PT to some extent (Mohring, 1972), and that optimization of frequency is more important than optimization of fare. However, the results differ in that the results in Börjesson et al. for Stockholm suggest that increased fares during peak hours would increase welfare, whereas the results from the present analysis of Uppsala suggests that reduced fares in the inner zone during peak hours would increase welfare.\textsuperscript{22} This difference between the studies may be due to a lower degree of crowding on buses combined with the fact that street congestion is not priced in Uppsala. That is, low PT fares may partly work as a second-best substitute for congestion charges. The sensitivity analysis with respect to congestion suggests that the congestion reduces the optimal prices by about 10–30 percent, which should be of similar order as the second-best effect in this respect.

The sensitivity analyses indicate that optimization of PT fares in an urban setting is sensitive to the basic assumptions about the MCPF and the social costs and benefits of parking. It is crucial to have good data on the cost and benefits of parking, and there is currently a lack of studies in this area. An interesting expansion of the present study would be to examine what implications an optimization of parking fees would have for optimal PT fares.

\textsuperscript{22} Basso and Silva (2014) also found it optimal to reduce fares for the larger cities London and Santiago (Chile)
In the present study, two welfare optima with respect to supply and fares have been analyzed, one unrestricted welfare optimum and one given the restriction that the aggregate generalized cost should not increase for any OD pair, time period, and mode combination. Both optima involve a reduction in supply and fares. Of these two, the Pareto scenario may seem more attractive as most of the large total gains in the welfare optimum can be achieved without any consumer group being worse off in terms of generalized travel cost than in the baseline scenario. Also, in the Pareto scenario, the total number of PT trips increases substantially (and there are also some environmental benefits). We can see that from an environmental perspective, it is more effective to decrease fares than to increase supply. This is because the fare reduction attracts travelers to the PT mode and hence reduces the emissions from the cars, without increasing the emissions from buses. Here there is obviously a tradeoff between crowding on buses and the environmental benefits. In the welfare optimum, this tradeoff is balanced with the welfare weights implied by the shadow prices in the national guidelines. However, the tradeoff does not consider welfare distribution.

Another point that we think is general is that when policy is at an optimum, the model prescribes that an increase in supply should be accompanied with a decrease in fares. This is because an increase in supply means spare capacity that should optimally be utilized, so it is a welfare gain to increase the effort to attract consumers.23

The model may underestimate benefits of proposed changes, as it does not consider the benefits of reduced congestion for walkers and cyclists24 and passengers on long-distance buses with an origin or destination in the city center. This also means that the benefits of transfers to these modes may be underestimated, which in turn means that reductions in congestion and crowding on buses may be underestimated, too. However, there is one source of bias in the opposite direction, namely that there may be a rebound effect of car traffic routing, which is not modeled. This arises if some O-O car traffic switches from going around the center to going through the center when congestion is reduced. Also, we have not considered that the overall travel demand may increase if the generalized costs of trips are reduced, which is another rebound effect. However, since the effects on delays are rather modest in the examined policy scenarios, these biases are probably rather small.

Increased running times due to increased boarding times (boarding times increase with number of passengers per bus) are not considered, which affects both street congestion and in-vehicle times of PT. In addition, it may be that the model underestimates the benefits of spare capacity in rush hours. This is because demand is variable and not evenly spread across departures and patterns in occupancy may differ across routes compared to Figure 2. This means that although there is no crowding in the model in the baseline scenario, it may occur if the demand varies considerably. For example, it may be valuable to

23 This observation is parallel to the Mohring effect meaning that the welfare from a purely commercial operation in many cases can be improved upon by reducing fares and increasing frequency
24 The positive external health effects of additional exercise are not included either.
distinguish between the opposite directions of trips during the morning and afternoon peaks in future work.
REFERENCES


Kilani, Proost, van der Loo (2014). "Road pricing and public transport pricing reform in Paris: Complements or substitutes?”, Economics of Transportation, 3, 175-187


Swedish Transport Administration, (2015). Handbok för vägtrafikens luftföroringar, Bilaga 6.1. Available at:
socially optimal fares and frequencies for urban bus services in small cities

http://www.trafikverket.se/TrvSeFiler/Fillistningar/handbok_for_vagtr
afikens_luftfororeningar/kapitel_6-bilagor_emissionsfaktorer.pdf

Swedish Transport Administration, (2016a). Analysmetod och
samhällsekonominiska kalkylvärden för transportsektorn: ASEK 6.0.

Swedish Transport Administration (2016b). Prognos för persontrafiken 2040 -

costs of light rail, heavy rail and bus rapid transit over a radial public
transport network”. Research in Transportation Economics 29, 231-242

Trafikanalys (2016). Swedish national and international road goods transport
2015, official statistics.


Uppsala län – Kollektivtrafikförvaltningen UL.

Uppsala municipality (2016). Resvaneundersökning hösten 2015 – En
kartläggning av kommuninvånarnas resmönster hösten 2015.

utilities: pricing in urban and suburban transportation”, American
Economic Review, LIII (2), pp. 452-465

motorists’ valuations of travel time with traffic conditions”
Transportation Research Part A, 46:213–225
APPENDIX A: A QUADRATIC UTILITY FUNCTION

Consider the following utility function of a representative consumer; additivity separable, quadratic in the quantity of each travel option, \( q_i \), and linear in other generalized consumption, \( c \), (including leisure), as in Börjesson et al. (2017):

\[
    u = c + \sum_i [a_i q_i - 0.5b_i q_i^2] \\
    c = y + p_L L - \sum_i p_i q_i \\
    q = \sum_i q_i
\]

where \( y \) is income, \( p_L \) is the price of leisure, \( L \) is leisure, \( p_i \) is generalized cost of option \( i \) and \( q \) is total travel demand. \( a_i, b_i, y, p_L, L, q \) are constant, \( p_i \) are non-choosable variables, and \( q_i \) are optimization variables for the representative individual.

When \( u \) is maximized subject to \( q_i \), the following holds:

\[
    \frac{\partial u}{\partial q_j} = -p_j - \sum_i [a_i - b_i q_i] = 0 
\]

(A2)

Differentiating with respect to \( p_i \) gives (after simplification):

\[
    \frac{\partial q_j}{\partial p_j} = -1 - \sum_i [b_i] 
\]

(A3)

B3 implies:

i. \( \frac{\partial q_j}{\partial p_j} \) is constant, i.e. a linear demand function.

ii. \( \frac{\partial q_j}{\partial p_j} = \frac{\partial q_i}{\partial p_i} \) for all \( i, j \).

i. is consistent with demand modeling and consumer surplus calculation (rule of one-half) in BUOV. However, ii. may be too restrictive in practice.
APPENDIX B: NOMENCLATURE

\( A_z \)  Area of zone (Inner, outer)

CS  Consumer surplus

CT  Congestion costs for trucks

\( D \)  No. of trips - Demand

\( d \)  Distance (OD, travel distance, route)

\( \delta \)  Delay (percentage)

\( c \)  Cost of supply (per km, per hour, per capacity (capital cost), in total)

SC  Seating capacity of each bus

DC  Distance cost of car (inner, outer, fixed)

\( E \)  Total net external effect, excluding congestion (PT, car; inner; outer)

\( e \)  Net external effect per vehicle km, excluding congestion (PT, car; inner; outer)

\( \varepsilon \)  Elasticity

\( f \)  Frequency

\( FC \)  Fixed cost of trip (parking fees, fixed cost of changes of vehicles, walking time)

GC  Generalized consumer cost/trip

\( o \)  Occupancy (mean, point; car, PT)

\( p \)  Fare (PT), Parking fees (car)

PS  Producer surplus

PR  Parking revenue

\( Q \)  Flows (Person, vehicle per hour; eq. per area and hour)

\( r \)  Revenue from fares

\( S \)  Supply (departures per hour)

\( T_{TP} \)  Length of period (Peak, OP)

\( t \)  Travel time (In-vehicle time (IVT), changing time, wait at home)

\( \tau \)  Marginal cost of public funds, MCPF

\( VoT \)  Value of time (IVT, changing time, wait at home, trucks, PT-supply)

\( W \)  Total welfare

\( \gamma \)  Level of subsidy (result)

\( \Psi \)  Set of restrictions for welfare optima

Super- and sub-scripts

Modes (m): car, PT (Public transport), WB (Walking/Bicycle)

Zones (z): Inner = I, Outer = O

OD pairs (OD): I-I, I-O, O-O

Locations (L): Center = C, Border inner = BI, Border outer = BO, Terminus = T

Time period (TP): P (peak), OP (off-peak)

Model iteration (MI): i

Flow type: pers = person, v = vehicle

28
APPENDIX C: USER EQUILIBRIUM

Beckman et al. (1956) showed that in a road network, a user equilibrium exists if the flows on all roads resulting from congestion costs are the same as the flows that produce these same costs. In the model in the present paper, the road and line networks are not represented. However, the choice of mode and timing is analogous to the routing choice in the network problem in Beckman et al. The analogues required condition is that the trips on all travel alternatives resulting from crowding and congestion costs, are the same as the trips that produce these same costs, which is met, although in a heterogeneous way.\textsuperscript{25}

The per-hour person-distance flow in each mode, per zone and time period (in iteration \( i \)) is:

\[
Q_{z,TP,i}^{\text{pers,m}} = \frac{\sum_{OD} D_{OD,m,TP,i} \gamma_{z,OD}^x d_{OD}}{T_{TP}},
\]

where \( D_{OD,m,TP,i} \) is the number of demanded trips for each travel alternative for each OD pair (in iteration \( i \)), \( \gamma_{z,OD}^x \) is the mean fraction of each trip per OD pair that goes through zone \( z \), \( d_{OD} \) is the traveled mean distance for each OD pair, and \( T_{TP} \) is the length of each time period. The superscript \( \text{pers} \) in eq. (C2) denotes person and next in eq. (3) the superscript \( v \) denotes vehicle.

The vehicle-distance flow of PT per area and hour in each zone and time period is:

\[
Q_{z,TP}^{v,PT} = \frac{S_{z,TP} d_z^i}{A_z},
\]

where \( S_{z,TP} \) is the bus supply in departures per hour in zone \( z \) and time period \( TP \), \( d_z^i \) is the mean length of each route in each zone, and \( A_z \) is the area size of zone \( z \).

The vehicle flow of cars per area and hour in each zone and time period is:

\[
Q_{z,TP,i}^{v,car} = \frac{Q_{z,TP,i}^{\text{pers,car}}}{A_z o_{car}},
\]

where \( o_{car} \) is the occupancy per car (including drivers and passengers).

The total vehicle-equivalent flow per area and hour in each zone and time period is:

\[
Q_{z,TP,i}^v = (Q_{z,TP}^{v,PT} + Q_{z,TP}^{v,fr,ght}) \cdot \rho + Q_{z,TP,i}^{v,car}.
\]

\textsuperscript{25} In the numerical part of the paper, a robustness check of the user equilibrium (in welfare optimum) has been performed, where the number of switchers in the first iteration has been multiplied by a fraction. Three different fractions have been tested: 0.2, 0.5, and 0.8. These changes in model specification yield no change in the last iteration (user equilibrium), indicating that the user equilibrium is robust and unique.

\textsuperscript{26} Normally, \( \gamma_{O,O}^x \) is 0 or 1. However, for O-O trips it is most reasonable to assume that a fraction goes through the city center, i.e., the inner zone.
where $Q_{z,TP}^{v,f} \text{ freight}$ is the relevant flow of trucks for freight purposes (static demand) and $\rho$ indicates how much congestion a bus or truck generates compared with a car.

The total percental delay per trip compared with free flow conditions in each zone and time period is:

$$\delta_{z,TP,i} = \alpha \cdot Q_{z,TP}^v + \beta \cdot (Q_{z,TP}^v)^2,$$  \hspace{1cm} (C6)

where $\alpha$ and $\beta$ are calibration parameters.

The in-vehicle travel time (for each OD-pair, in each zone, for car and PT modes in each time period and iteration) is:

$$t_{OD,z,TP,i}^{\text{int, m}} = t_{OD,z,TP,i}^{\text{int, m}} \cdot \frac{1 + \delta_{z,TP,MI=0}}{1 + \delta_{z,TP,i}},$$  \hspace{1cm} (C7)

where $m \in \{ \text{car}, \text{PT} \}$ and $\text{MI} = 0$ means baseline ($\text{MI} = \text{model iteration}$)

The mean bus occupancy rate for each zone and time period in iteration $i$ is:

$$\bar{o}_{z,TP,i}^\text{PT} = \frac{Q_{z,TP,i}^\text{PT}}{Q_{z,TP,i}^v}.$$  \hspace{1cm} (C8)

The point occupancy rate at the terminus in each time period and iteration is:

$$o_{TP,i}^\text{PT,T} = 0.$$  \hspace{1cm} (C9)

The point occupancy rate in the outer zone at border in each time period and iteration is:

$$o_{TP,i}^\text{PT,BO} = o_{TP,i}^\text{PT} \cdot 2.$$  \hspace{1cm} (C10)

The point occupancy rate in the inner zone at border in each time period and iteration is:

$$o_{TP,i}^\text{PT,BI} = o_{TP,i}^\text{PT,BO} \cdot \frac{S_{z,TP,i}}{S_{z,TP}}.$$  \hspace{1cm} (C11)

The point occupancy rate at the center in each time period and iteration is:

$$o_{TP,i}^\text{PT,C} = 2 \cdot o_{TP,i}^\text{PT} - o_{TP,i}^\text{PT,BO}.$$  \hspace{1cm} (C12)

The in-vehicle value of time for car in each zone, time period, and iteration is assumed to increase with congestion level:

$$V_{z,TP}^{\text{int, car}} = (1 + \omega \cdot \delta_{z,TP,i}) \cdot V_{z,TP}^{\text{int, car}},$$  \hspace{1cm} (C13)
where $\omega > 0$ is a parameter indicating how VoT increases with increased congestion$^{27}$ and $VoT_{free}^{int,car}$ is the free flow (in vehicle) value of time.

The in-vehicle value of time for PT is assumed to be piecewise linear in occupation, with a kink at 100 percent occupancy (the threshold above which people need to travel standing up).$^{28}$ The in-vehicle value of time for PT in each location, time period, and iteration is:

$$VoT_{int,PT}^{TP,i} = \begin{cases} \sigma_1 \cdot o_{TP,i}^{PT} + VoT_{o=0}^{int,PT} & \text{if } [o_{TP,i}^{PT} \leq 1] \\ \sigma_2 \cdot o_{TP,i}^{PT} + VoT_{o=1}^{int,PT} & \text{if } [o_{TP,i}^{PT} > 1] \end{cases}$$

(C14)

where $\sigma_1, \sigma_2$ are parameters indicating the slopes, and $\sigma_2 > \sigma_1 > 0$. $VoT_{o=0}^{int,PT}$ and $VoT_{o=1}^{int,PT}$ are the values of time for 0% and 100% (of number of seats) occupancy respectively.

The in-vehicle value of time for PT in the outer zone in each time period and iteration is:

$$VoT_{o,TP,i}^{int,PT} = \frac{VoT_{int,PT}^{TP,i} + VoT_{int,PT}^{TP,i}}{2}$$

(C15)

The in-vehicle value of time for PT in the inner zone in each time period and iteration is:

$$VoT_{i,TP,i}^{int,PT} = \frac{VoT_{int,PT}^{TP,i} + VoT_{int,PT}^{TP,i}}{2}$$

(C16)

The generalized consumer cost per car trip in each OD pair, time period, and iteration is:

$$GC_{OD,TP,i}^{car} = \frac{DC \cdot d_{OD} + p_{OD,TP}^{car}}{o_{car}} + \sum_z (VoT_{z,TP,i}^{int,car} \cdot t_{OD,z,TP,i}^{int,car})$$

(C17)

where $DC$ is the distance cost per car (including capital, fuel and wear and tear), $p_{OD,TP}^{car}$ is the mean parking fee paid per car, OD pair and time period.

The generalized consumer cost per PT trip in each zone, time period, and iteration is:

$$GC_{OD,TP,i}^{PT} = p_{OD,TP}^{PT} + \sum_{j=wait, walk, ch} (VoT_{T,j}^{int,PT} \cdot t_{j,OD,TP,T}^{int,PT}) + \sum_z (VoT_{z,TP,i}^{int,PT} \cdot t_{OD,z,TP,i}^{int,PT})$$

(C18)

where $wait$ denotes the waiting time, $walk$ walking time connecting to the bus stops, and $ch$ changing time between bus routes for each trip.

$^{27}$ Due to more concentration demanding driving and irritation, based on Wardman and Ibáñez (2010).

$^{28}$ This functional form is from the Swedish national guidelines for cost benefit analysis (Swedish Transport Administration, 2016a)
The change in number of trips per mode, OD pair, and time period due to a policy reform is (in iteration $i$):

$$
\Delta D^\text{tot}_{OD,m,TP,i} = \Delta \tilde{D}_{OD,m,TP,i} + \sum_{m,TP} (-\Delta \tilde{D}_{OD,m,TP,i} \cdot \theta^{OD,m,TP}_{m,TP})
$$

(C19)

$$
\Delta \tilde{D}_{OD,m,TP,i} = \Delta G^m_{OD,TP,i} \cdot \varepsilon_{m,TP}
$$

(C20)

is the partial change in demand resulting from changes in own generalized cost of each travel alternative $(m, TP)$.

$$
\Delta G^m_{OD,TP,i} = G^m_{OD,TP,i} - G^m_{OD,TP,0}
$$

(C21)

$\varepsilon_{m,TP}$ is the own generalized cost elasticity (which is derived from the own price elasticity$^{29}$).

$\theta^{OD,m,TP}_{m,TP}$ is the share of changes in trips in one alternative $(m, TP)$ that is a result of changes in the generalized cost in another alternative $(\bar{m}, \bar{TP})$, defined as:

$$
\theta^{OD,m,TP}_{m,TP} = \frac{\phi^{OD,m,TP}_{m,TP} - D_{OD,m,TP,0}}{\sum_{m,TP} (\phi^{OD,m,TP}_{m,TP} - D_{OD,m,TP,0})^l}
$$

(C22)

where $\phi^{OD,m,TP}_{m,TP}$ is a dummy variable denoting the travel alternatives closest to the current one, defined as:

$$
\phi^{OD,m,TP}_{m,TP} = \begin{cases} 
1 & \text{if } \bar{m} = m, \bar{TP} \neq TP \\
1 & \text{else if } \bar{m} = m, \bar{TP} = TP \\
0 & \text{else}
\end{cases}
$$

(C23)

That is, the distribution of moves of trips to the three closest alternatives is proportional$^{30}$ to number of trips in each of these three alternatives in the baseline scenario.

As a last step, the total number of trips for each travel alternative within each OD pair is updated as:

$$
D_{OD,m,TP,i+1} = D_{OD,m,TP,i} + \Delta D^{tot}_{OD,m,TP,i}
$$

(C24)

Eqs. (2–24) are run in a recursive loop (where $i$ is increased by 1 for each iteration) until the system reaches the user equilibrium; that is, the first iteration

$^{29} \varepsilon_{m,TP} = \frac{\text{price}}{\text{OD}} \cdot \frac{\sum_{m,TP} (\Delta G^m_{OD,TP,i} \cdot \phi^{OD,m,TP}_{m,TP})}{\sum_{m,TP} (\Delta G^m_{OD,TP,i} \cdot D_{OD,m,TP,0})}$

$^{30}$ The proportional assumption could be categorized as a naive assumption, due to lack of data. It may be reasonable when comparing modes during the same time period, but may be problematic when comparing mode choice with timing choice. The model can easily be updated in this regard as new data on the subject becomes available.
when there is no substantial difference between any variable compared with in the previous iteration.

**APPENDIX D: AGGREGATE WELFARE**

The change in consumer surplus (due to a policy change) compared with baseline is defined by the rule of one-half, for each mode$^{31}$, time period and OD pair (in iteration $i$), as:

$$
\Delta CS_{OD,m,TP,i} = -\Delta GC_{OD,TP,i}^m \cdot \frac{D_{OD,m,TP,i} \cdot D_{OD,m,TP,0}}{2}.
$$

The total change in consumer surplus (in iteration $i$) compared with baseline is defined as:

$$
\Delta CS_i = \sum_{OD,m,TP} (\Delta CS_{OD,m,TP,i}).
$$

The total cost of providing supply in each zone and time period is:

$$
c_{z,TP,i} = \left( c^d \cdot d^l_z + \left( c^t + k_{TP,i} \cdot c^k \right) \cdot (1 + \delta_{z,TP,i}) \cdot t^i \right) \cdot \theta_z \cdot S_{z,TP} \cdot T_{TP},
$$

where $t^i$ is the mean time of each one-way trip, $c^d$ is cost per distance for bus (fuel and wear and tear), $c^t$ is the running cost per hour for bus (wage of drivers), $c^k$ is the capital cost per hour for bus, and $\theta_z$ is the share of the route that is in zone $z$. $k_{TP,i}$ is a parameter indicating the capital cost share:

$$
k_{TP,i} \in \begin{cases} 
1 & \text{if } \left[ \sum_z \left( (1 + \delta_{z,TP,i}) \cdot t^i \cdot \theta_z \cdot S_{z,TP} \cdot T_{TP} \right) > 0.5 \right] \\
0.5 & \text{if } \left[ \sum_z \left( (1 + \delta_{z,TP,i}) \cdot t^i \cdot \theta_z \cdot S_{z,TP} \cdot T_{TP} \right) = 0.5 \right] \\
0 & \text{else}
\end{cases}
$$

The total cost of supply is:

$$
c_i = \sum_{z,TP} c_{z,TP,i}.
$$

The total revenue from PT fares is:

$$
r_i = \sum_{OD,TP} (p_{OD,TP}^{PT} \cdot D_{OD,TP,i}^{PT}),
$$

where $p_{OD,TP}^{PT}$ is the fare (for each PT trip) for each OD pair in each time period.

The (with subsidy negative) producer surplus is:

$$
PS_i = r_i - c_i.
$$

---

$^{31}$ However, there is no change in generalized costs for walking/bicycle, so in practice this calculation is performed for car and bus only.

33
The level of subsidy is:

\[ \gamma = \frac{1 - r_i}{c_i}. \]  

(D32)

It is assumed that parking is marginally cost based; that is, the total net benefit from changes in parking revenues is:

\[ \Delta P R_i = 0. \]  

(D33)

The congestion benefits for trucks from a policy change are defined as:

\[ \Delta CT_i = \sum_{z,TP} \left( D_{z,freight,TP} \cdot t_{z,freight,TP} \cdot \left[ \delta_{z,TP,0} - \delta_{z,TP,i} \right] \right) \]  

(D34)

Total net external effect\(^{32}\) from person transport vehicles (excluding congestion but including internalization from taxes) is defined as:

\[ E_i = \sum_{z,TP} \left( q_{z,TP,i}^{pers,car} \cdot o_{car} \cdot e_{z,car} + S_{z,TP} \cdot d_z \cdot e_{z,PT} \right) \]  

(D35)

where \( e_{z,car} \) and \( e_{z,PT} \) are the net of valuation of environmental damages minus taxation per vehicle in each zone for car and bus, respectively.

The total welfare effects of a given policy change is:

\[ \Delta W_i = \Delta C S_i + (1 + \tau) \cdot (\Delta P S_i + \Delta P R_i) + \Delta C T_i + \Delta E_i, \]  

(D36)

where \( \tau \) is the MCPF, \( \Delta P S_i \) and \( \Delta E_i \) denote changes in producer surplus and the net social cost of external effects, compared with baseline.\(^{33}\)

The welfare optima given different restrictions are defined as:

\[ \max_{S, f}(\Delta W_i | \xi, \Psi), \]  

(D37)

where \( S \) and \( f \) are two-dimensional matrices, \( \xi, \Psi \) are a set of restrictions, and \( i * \) denotes the user equilibrium.\(^{34}\) \( S \) has the dimensions time period and zone, while \( f \) has the dimensions time period and OD pair.

\[ \xi \in \left\{ \begin{array}{l} p_{i-1,TP}^{PT} \geq p_{i-1,TP}^{PT} \quad \text{for each time period, } TP^* \\ p_{i-1,TP}^{PT} \geq p_{0-1,TP}^{PT} \end{array} \right\} \]

Thus, within each time period, it should not be cheaper to travel in both zones than in just one.

\(^{32}\) Including effects on traffic safety, noise, and emissions.

\(^{33}\) It is assumed that all parking is publicly owned, which is a simplification.

\(^{34}\) Note that eq. (37) implies that the policy maker is a Stackelberg leader, setting policy to in anticipation of what the total response (in the last iteration only) will be. That is policy is set one time only and not in every iteration \( i \).
Socially optimal fares and frequencies for urban bus services in small cities

\[ \Psi \in \emptyset \] defines the welfare optimum.
\[ \Psi \in \{ S = S^0 \} \] defines the optimum given fixed supply.
\[ \Psi \in \{ p = p^0 \} \] defines the optimum given fixed fares.
\[ \Psi \in \{ \Delta G_{OD,TP,i}^m \leq 0 \} \] for all \( OD, m, TP \), defines the optimum given the restriction of Pareto improvements from the baseline (called Pareto scenario).
APPENDIX E: DATA

<table>
<thead>
<tr>
<th>OD pair</th>
<th>Walking time PT (min)</th>
<th>No. of bus changes</th>
<th>IVT PT (min)</th>
<th>IVT Car (min)</th>
<th>Distance mean (km)</th>
<th>Distance car (km)</th>
<th>Distance PT (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-I</td>
<td>7.2</td>
<td>0.13</td>
<td>4.0</td>
<td>4.1</td>
<td>2.1</td>
<td>2.2</td>
<td>2.4</td>
</tr>
<tr>
<td>I-O</td>
<td>9.7</td>
<td>0.35</td>
<td>13.6</td>
<td>7.6</td>
<td>4.8</td>
<td>5.1</td>
<td>5.4</td>
</tr>
<tr>
<td>O-O</td>
<td>11.8</td>
<td>0.71</td>
<td>19.6</td>
<td>7.8</td>
<td>4.9</td>
<td>6.0</td>
<td>7.2</td>
</tr>
</tbody>
</table>

Table E1: SAMPERS data used

In Table E2 various peak versus OP assumptions are reported. A peak of five hours, is based on documentation on SAMPERS, and according to reported statistics for Uppsala 2010, public transport is open for 19 hours which leaves 14 hours to the OP). Headways are based on glancing through the time tables of 2016.

<table>
<thead>
<tr>
<th></th>
<th>Number of hours</th>
<th>Baseline headway</th>
<th>Parking cost* I-O</th>
<th>Parking cost* I-I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak</td>
<td>5</td>
<td>10</td>
<td>60 SEK</td>
<td>3 SEK</td>
</tr>
<tr>
<td>OP</td>
<td>14</td>
<td>15</td>
<td>30 SEK</td>
<td>3 SEK</td>
</tr>
</tbody>
</table>

Table E2: Peak versus OP assumptions (based on various information sources)

*One-way, crude estimates based on local parking regulations. The I-I costs are based on the assumption that residents park at discounted prices.

In table E3, additional data used are reported.
<table>
<thead>
<tr>
<th>Source</th>
<th>Parameter/data</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reported statistics for Uppsala 2010</strong></td>
<td>Executed no. of bus-hours</td>
<td>472 340</td>
</tr>
<tr>
<td></td>
<td>Executed no. of bus-km</td>
<td>9 100 000</td>
</tr>
<tr>
<td></td>
<td>No. of bus-hours at terminus (slack)</td>
<td>85 000</td>
</tr>
<tr>
<td></td>
<td>Fare</td>
<td>11.2 SEK</td>
</tr>
<tr>
<td><strong>SKL, 2014</strong></td>
<td>Mean point occupancy in the base</td>
<td>8.5</td>
</tr>
<tr>
<td><strong>ASEK 6 (Swedish transport administration, 2016a, (regional trips 2014))</strong></td>
<td>MCPF</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>IVT Car on empty street</td>
<td>91.6 SEK/h</td>
</tr>
<tr>
<td></td>
<td>Increased IVT car for a doubling of travel time*</td>
<td>+1/3</td>
</tr>
<tr>
<td></td>
<td>IVT PT in empty bus</td>
<td>33.9 SEK/h</td>
</tr>
<tr>
<td></td>
<td>IVT PT increase for 100% occupancy</td>
<td>+18.0 SEK/h</td>
</tr>
<tr>
<td></td>
<td>IVT PT increase for 200% occupancy (compared to 100% occupancy)</td>
<td>+27.0 SEK/h</td>
</tr>
<tr>
<td></td>
<td>Value of increased frequency (decreased waiting time)</td>
<td>41.6 SEK/h</td>
</tr>
<tr>
<td></td>
<td>Change of vehicle time (also applies for walking time in present study)</td>
<td>110 SEK/h</td>
</tr>
<tr>
<td></td>
<td>Occupancy car</td>
<td>1.53</td>
</tr>
<tr>
<td><strong>Combination of Trafikanalyser 2016 and Swedish transport administration (2016b)</strong></td>
<td>Share trucks (of cars)</td>
<td>2.6%</td>
</tr>
<tr>
<td><strong>Börjeson et al., 2017</strong></td>
<td>Km-cost for car</td>
<td>1.5 SEK</td>
</tr>
<tr>
<td></td>
<td>Buss equivalent to number of car (congestion)</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>Number of weekdays a year</td>
<td>250</td>
</tr>
<tr>
<td><strong>Assumption</strong></td>
<td>Number of seats in one bus</td>
<td>30</td>
</tr>
</tbody>
</table>

Table E3: Other parameter values and utilized data
*Estimated in present study to match ASEK recommendations, based visual interpretation on figure in underling study, Wardman and Ibáñez (2010).
APPENDIX F: CALIBRATION

Travel with start or destination outside the city center Uppsala (which has a destination or starting point in Uppsala) is analyzed with different methods, depending on the assumptions on how they interact with traffic. For the public transport travelers who come from outside the analyzed zones, they are assumed to arrive by train or regional bus and arrive at 100 percent to Uppsala main station, from which they proceed to the respective target point by bus or walking, according to the distribution of these two modes for the respective zone type (Inner/Inter). Then this part of the trip is added to the total travel by Bus/Walking for the respective zone (Inner/Inter). This is because these travelers are likely to consider Bus/Walk/Taxi if the cost picture changes. Even though the elasticities probably differ somewhat from other travelers (especially for the car option), because these trips relatively few, this assumption is not likely to affect the results much.

For the car journeys from outside of Uppsala to the outer zone (of Uppsala) it is assumed that these are not affected by changes in public transport and therefore they are excluded from the analysis, as well as all walking and bike trips travelling from Uppsala to the outside of Uppsala. The cars traveling between outside Uppsala and the inner parts of Uppsala both experience and induce congestion and should therefore be included in the congestion and welfare calculations. For simplicity, therefore these car trips are added to the inter trips although their true individual elasticities differ from the ones that go the shorter distance between the inner and outer zones within Uppsala (due to different generalized cost per trip). This means that the own price elasticities for car inter trips may be somewhat overestimated.

The travel distances differ among modes in baseline (not much in the inner zone, but for the outer zone walking/biking trips are substantially shorter than are car and public transport trips). Because of this some adjustments of the model with regard to travel distance is needed. A first step is to recognize that the mean travel distances hides a large degree of within mode heterogeneity. A reasonable assumption is that the walkers and cyclist that are most likely to switch to another mode is the ones that have a trip length that is in the upper part of the distribution of trip lengths among walkers and cyclist, that is closer to the average of public transport and car users. In the same time, the public transport and car users that are most likely to switch mode to walking or bicycling is the ones that have trip lengths that are shorter than the average for car and public transport users, that is closer to the average for walkers and cyclists. Because of this, in combination with the tractability of simplicity, all switchers of mode or of time period are assumed to have a trip length that is the same across modes (sample mean). Therefore, for each iteration total vehicle distance is updated (from baseline) by adding/subtracting the distance from the switching trips according to this principle. However, the distance also shows up in the calculation of generalized cost of the car alternative (eq. (17)). Because elasticities are based on the costs of the whole sample, not just the switchers, distances in eq. (17) are based on mean distances for car users only. This is also important when calculating the summed-up welfare effects of decreased congestion.
It is assumed that the parts of the inter-journeys that are in the inner zone are approximately as long as the length of trips within the inner zone, i.e. about 2 km. Based the assumptions for Figure 1 & 2, the distance to the point of $B$ (the border between inner and outer zone) is calculated to be 16 percent of the distance from $C$ to $T$.

In the volume delay function in eq. 6, the calibrated parameters are $\alpha = 8.18$ and $\beta = 1.12$ (per k vehicle-equivalent-km/h/km$^2$). Since $Q_{z,T}^v$ are in the order of ~1000 vehicle-equivalent-km/h/km$^2$, this means that the quadratic component) is about 700 times as important as the linear component, so that $\alpha$ could have been approximated to zero.
APPENDIX G: MARGINAL RESULTS

Table G1 shows the effects on number of trips, congestion (delay, both for car and public transport users) and crowding (occupancy) of a marginal increase in bus frequency. From Table G1, one can see that the estimated number of attracted travelers from a supply increase is not massive35 (the elasticity is at most 0.38), even though both the effects on increased frequency and on reduced crowding are considered. This is because crowding is benign in the baseline, and even though the cost associated with waiting is considerable, there are other cost components that are important too - walking time to stations, travel time and fares. This means that waiting time per se is only a fraction of the generalized cost of a public transport trip. As a consequence, the rebound effect on crowding is modest so the effect on crowding is rather large with elasticities up to 0.80. In contrast, effects on the congestion is rather small.

Table G2 shows the effects on number of trips, congestion (in terms of delay) and crowding (in terms of occupancy) of differentiated marginal increases in fares. From Table G2, one can see that the effect on attractiveness of public transportation for the inner city is greater for changes in fares (elasticities about -0.50) than changes in supply. The reason for this is that for the inner city, trips are substantially shorter than for the O-O and Inter trips, which means that the generalized cost for I-I is much lower, which in turns means that the fare constitutes a larger share of the generalized costs. Supply changes, however, affect also inter and O-O trips (the effects on demand from supply adjustments is distributed over 2-3 OD-pairs). The effect on crowding is larger for the fare changes on inter trips than for fare changes within zones. This stems from a combination of three factors. First and foremost, the inter trips are more in numbers in the baseline than for each of the other two categories. Second, they are much longer than the I-I trips; and third, the have higher elasticity than the O-O trips. Again, we can see that the effects on delays are modest.

35 However, these elasticities are large compared to in to the supply elasticity of only 0.03 in Pyddoke et al. (2017).
## Socially optimal fares and frequencies for urban bus services in small cities

<table>
<thead>
<tr>
<th>Zone/OD</th>
<th>Time period</th>
<th>Mode</th>
<th>Baseline</th>
<th>Supply increase 1% (change as share of Baseline)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Inner Peak</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.00%</td>
</tr>
<tr>
<td>1-I</td>
<td>Peak</td>
<td>Car</td>
<td>6 484</td>
<td>-0.07%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PT</td>
<td>3 302</td>
<td>0.16%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Walk/Cycle</td>
<td>19 075</td>
<td>-0.01%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OP</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Car</td>
<td>12 042</td>
<td>0.01%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PT</td>
<td>4 752</td>
<td>-0.02%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Walk/Cycle</td>
<td>28 613</td>
<td>0.00%</td>
</tr>
<tr>
<td>Inter</td>
<td>Peak</td>
<td>Car</td>
<td>26 896</td>
<td>-0.03%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PT</td>
<td>12 015</td>
<td>0.08%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Walk/Cycle</td>
<td>23 191</td>
<td>-0.01%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OP</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Car</td>
<td>49 950</td>
<td>0.00%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PT</td>
<td>17 289</td>
<td>-0.01%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Walk/Cycle</td>
<td>34 786</td>
<td>0.00%</td>
</tr>
<tr>
<td>0-0</td>
<td>Peak</td>
<td>Car</td>
<td>13 478</td>
<td>-0.01%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PT</td>
<td>5 561</td>
<td>0.04%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Walk/Cycle</td>
<td>22 283</td>
<td>-0.01%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OP</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Car</td>
<td>25 030</td>
<td>0.00%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PT</td>
<td>8 002</td>
<td>0.00%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Walk/Cycle</td>
<td>33 424</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

### Delay (%)

<table>
<thead>
<tr>
<th>Zone/OD</th>
<th>Time period</th>
<th>Mode</th>
<th>Value</th>
<th>Inner Peak</th>
<th>OP Peak</th>
<th>Outer Peak</th>
<th>OP Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-I</td>
<td>Peak</td>
<td>Car+PT</td>
<td>95.5%</td>
<td>0.12%</td>
<td>0.01%</td>
<td>-0.01%</td>
<td>-0.01%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OP</td>
<td>41.9%</td>
<td>0.01%</td>
<td>0.09%</td>
<td>0.00%</td>
<td>-0.03%</td>
</tr>
</tbody>
</table>

### Mean occupancy (pers./vehicle)

| Zone/OD | Time period | Mode | Delay (%)
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Inner Peak</td>
</tr>
<tr>
<td>1-I</td>
<td>Peak</td>
<td>PT</td>
<td>62.3%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OP</td>
<td>48.0%</td>
</tr>
<tr>
<td></td>
<td>Peak</td>
<td>PT</td>
<td>27.4%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OP</td>
<td>21.1%</td>
</tr>
</tbody>
</table>

Table G1: Effects on demand, delay for car and occupancy in the buses from 1 percent increase in bus supply, in inner versus outer zone and in the peak versus the off-peak
### Table G2: Effects on demand, delay for car and occupancy in the buses from a 1 percent increase in fares, in inner versus outer zone and in the peak versus the off-peak

In Table G3 and Table G4, the welfare effects of marginal supply and fare adjustments are summarized. There appears to be an excess in both supply and fares in baseline (the only exception is fares for O-O in the peak). In some cases, this excess is large and the model suggests that there is socially beneficial to decrease supply as well as all but one of the fares (see the last rows of Table D3 and Table G4). Somewhat surprisingly, the model suggest that it is most socially desirable to decrease supply in the inner zone during the peak. This is because frequency is highest, cost of supply is largest and the model estimates large delays from congestion - and more buses on the streets worsen this situation. In the
same time, crowding in vehicles is not a large problem on average in baseline, not even in the inner zone during peak hours.  

<table>
<thead>
<tr>
<th>Supply adjusted for OD-pair Adjusted in time period</th>
<th>Inner</th>
<th>Outer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Peak</td>
<td>OP</td>
</tr>
<tr>
<td>Consumer surplus</td>
<td>+210</td>
<td>+878</td>
</tr>
<tr>
<td>Of which congestion benefits</td>
<td>-548</td>
<td>-366</td>
</tr>
<tr>
<td>Producer surplus, without MCPF</td>
<td>-1 624</td>
<td>-1 387</td>
</tr>
<tr>
<td>Congestion benefits for trucks</td>
<td>-13</td>
<td>-9</td>
</tr>
<tr>
<td>Other external benefits*</td>
<td>-4</td>
<td>-3</td>
</tr>
<tr>
<td>Of which CO₂ benefits**</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>Net social benefits</td>
<td>-1 918</td>
<td>-937</td>
</tr>
<tr>
<td>NSB as share of PS (%)***</td>
<td>-118%</td>
<td>-68%</td>
</tr>
</tbody>
</table>

Table G3: Social costs and benefits in SEK per week day of differentiated marginal (1%) increases in supply
* Net benefits of external effects and taxation of external effects
** Excluding internalization (negative sign means increased emissions).
*** "NSB as share of PS" denotes net social costs as share of the absolute value of the producer surplus, and is a measure of the social return of public spending.

<table>
<thead>
<tr>
<th>Fare adjusted for OD-pair Adjusted in time period</th>
<th>Inner</th>
<th>Inter</th>
<th>O-O</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Peak</td>
<td>OP</td>
<td>Peak</td>
</tr>
<tr>
<td>Consumer surplus</td>
<td>-395</td>
<td>-566</td>
<td>-1 802</td>
</tr>
<tr>
<td>Of which congestion benefits</td>
<td>-106</td>
<td>-119</td>
<td>-262</td>
</tr>
<tr>
<td>Producer surplus, without MCPF</td>
<td>+255</td>
<td>+225</td>
<td>+1 275</td>
</tr>
<tr>
<td>Congestion benefits for trucks</td>
<td>-1</td>
<td>-2</td>
<td>-5</td>
</tr>
<tr>
<td>Other external benefits</td>
<td>0</td>
<td>0</td>
<td>+1</td>
</tr>
<tr>
<td>Of which CO₂ benefits</td>
<td>0</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>Net social benefits</td>
<td>-65</td>
<td>-275</td>
<td>-149</td>
</tr>
<tr>
<td>NSB as share of PS (%)</td>
<td>-26%</td>
<td>-122%</td>
<td>-12%</td>
</tr>
</tbody>
</table>

Table G4: Social costs and benefits in SEK per week day of differentiated marginal (1%) increases in fares

---

Note that the direction of trips has not been taken into account, and this could be an important limitation for inter trips during the peak hour.

---

43
### APPENDIX H: SENSITIVITY RESULTS

In Table H1, the changes in demand in welfare optimum for sensitivity analyses are presented. The demand for public transport is considerably affected in the basic assumption tests.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Baseline optimum</th>
<th>Sensitivity scenario</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MCPF = 0</td>
<td>ΔPR 0.5</td>
<td>$\epsilon_{m,TP}$ = 0.4</td>
<td>$\overline{\alpha_{PT}}$ = 10</td>
<td>No delay</td>
<td>Value of frequency +50%</td>
<td></td>
</tr>
<tr>
<td>Car</td>
<td>+1%</td>
<td>-3%</td>
<td>+4%</td>
<td>+1%</td>
<td>+1%</td>
<td>+2%</td>
<td>+1%</td>
</tr>
<tr>
<td>PT</td>
<td>-3%</td>
<td>+20%</td>
<td>-21%</td>
<td>-3%</td>
<td>-5%</td>
<td>-10%</td>
<td>-2%</td>
</tr>
<tr>
<td>WB</td>
<td>+0%</td>
<td>-4%</td>
<td>+3%</td>
<td>+0%</td>
<td>+0%</td>
<td>+2%</td>
<td>-0%</td>
</tr>
</tbody>
</table>

Table H1: Changes in trips per mode in welfare optimum for sensitivity analyses.

In Table H2, the welfare results for sensitivity analyses are presented. The net social benefits of optimization are rather robust.

<table>
<thead>
<tr>
<th>Welfare component</th>
<th>Baseline optimum</th>
<th>MCPF = 0</th>
<th>ΔPR 0.5</th>
<th>$\epsilon_{m,TP}$ = 0.4</th>
<th>$\overline{\alpha_{PT}}$ = 10</th>
<th>No delay</th>
<th>Value of frequency +50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer surplus</td>
<td>-81 388</td>
<td>406 240</td>
<td>-359 521</td>
<td>-40 175</td>
<td>-98 304</td>
<td>-200 775</td>
<td>-49 375</td>
</tr>
<tr>
<td>Of which congestion benefits</td>
<td>24 049</td>
<td>56 288</td>
<td>-876</td>
<td>21 842</td>
<td>14 077</td>
<td>0</td>
<td>20 430</td>
</tr>
<tr>
<td>Producer surplus, without MCPF</td>
<td>325 662</td>
<td>-108 460</td>
<td>504 520</td>
<td>291 296</td>
<td>322 383</td>
<td>407 811</td>
<td>238 954</td>
</tr>
<tr>
<td>Parking revenues</td>
<td>0</td>
<td>0</td>
<td>91 504</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Congestion benefits for trucks</td>
<td>703</td>
<td>1 650</td>
<td>-35</td>
<td>632</td>
<td>685</td>
<td>0</td>
<td>589</td>
</tr>
<tr>
<td>Other external effects</td>
<td>1 350</td>
<td>873</td>
<td>1 694</td>
<td>1 266</td>
<td>1 412</td>
<td>1 516</td>
<td>1 208</td>
</tr>
<tr>
<td>Of which CO₂ benefits</td>
<td>180</td>
<td>986</td>
<td>-212</td>
<td>190</td>
<td>139</td>
<td>-83</td>
<td>203</td>
</tr>
<tr>
<td>Net social benefits (SEK)</td>
<td>344 024</td>
<td>300 302</td>
<td>416 970</td>
<td>340 407</td>
<td>322 890</td>
<td>330 895</td>
<td>263 062</td>
</tr>
</tbody>
</table>

Table H2: Welfare results for sensitivity analyses

44