A Note on the Stability of the Swedish Phillips Curve*

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Abstract

We use Bayesian techniques to estimate bivariate VAR models for Swedish unemployment rate and inflation. Employing quarterly data from 1995Q1 to 2017Q3 and new tools for model selection, we compare a model with time-varying parameters and stochastic volatility to a specification with constant parameters and covariance matrix. Regardless of whether we use price inflation or wage inflation, we find strong evidence in favour of the specification with time-varying parameters and stochastic volatility. Our results indicate that the Swedish Phillips curve has not been stable over time. However, our findings do not suggest that the Phillips curve has been flatter in more recent years.

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1. Introduction

Inflation in Sweden has been stubbornly low over the last years. This development is similar to that in several other inflation-targeting countries where inflation has been moderate, and increasing slowly, despite historically low policy-interest rates and developments in the real economy which many argue should have generated a stronger inflationary pressure; see, for example, Jansson (2017) and Yellen (2017). One explanation for the low inflation which has been put forward is that the Phillips curve has become flatter, for example due to globalisation and digitalisation.\(^1\)

In this paper we contribute to the discussion regarding the properties of the Phillips curve by providing evidence based on Swedish data. Employing a Bayesian VAR (BVAR) framework, we estimate bivariate models using quarterly data on unemployment rate and inflation. The models are estimated under two different assumptions concerning the dynamics and covariance matrix. As noted by a growing literature, time variation in both dynamics and volatility appears to be important features of macroeconomic relationships; see, for example, Cogley and Sargent (2005), Koop et al. (2009), Chan et al. (2016), Akram and Mumtaz (2017) and Knotek and Zaman (2017). We therefore estimate a model with time-varying parameters and stochastic volatility and compare this model to a traditional specification with constant parameters and covariance matrix. Relying on new tools for model selection developed by Chan and Eisenstat (2018), we formally assess which model is preferred by the data. This constitutes a step forward relative to the vast majority of previous related research. Since model selection is a non-trivial issue when models with time-varying parameters and stochastic volatility are involved, it has often simply been assumed that it is reasonable to employ a model with such features.\(^2\) In this paper we instead evaluate this assumption using marginal likelihoods and Bayes factors and can provide statistical evidence on the stability of the Swedish Phillips curve.

The models are first estimated using price inflation seeing that this has been the general focus of the debate regarding potential flattening of the Phillips curve. It is also of primary interest from a Swedish perspective since the Riksbank is an inflation-targeting central bank with a target variable based on an index of consumer prices. However, we also conduct analysis based on wage inflation. This has the benefit of taking our analysis in the direction of Phillips’ (1958) original observation (which was based on wage inflation). But perhaps more importantly, it also means that we can scrutinise what may be described as a key relationship through which central banks assume that their goals are achieved: Expansive monetary policy is assumed to stimulate the real economy, thereby lowering the unemployment rate; this tighter labour market pushes up

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\(^1\) For two recent contributions to the debate regarding the potentially flattened Phillips curve, see Blanchard et al. (2015) and Coibion and Gorodnichenko (2015). Another argument in favour of a flattened Phillips curve is the "missing disinflation" in the aftermath of the global financial crisis of 2008 as discussed by, for example, the IMF (2013).

\(^2\) Exceptions do exist; see, for example, Koop et al. (2009) and Karlsson and Österholm (2018). In addition, studies such as D’Agostino et al. (2013) and Barnett et al. (2014) have relied on out-of-sample predictive criteria to distinguish between models.
wage growth (wage inflation) which in turn increases price inflation. Assessing the stability of the relation between the unemployment rate and wage inflation could accordingly be an informative exercise.

Our results show that both time-varying parameters and stochastic volatility are relevant features when modelling the data, regardless of whether prices or wages are used to calculate the inflation measure. This indicates that the Swedish Phillips curve has not been stable over time. However, we do not find any evidence suggesting that the Phillips curve has been flatter than usual during the last years of our sample and accordingly conclude that the low inflation in the last few years has not been caused by a changing slope of the Phillips curve. Our findings instead suggest that low trend inflation may have contributed to this development.

The rest of this paper is organised as follows: In Section 2, we describe the Bayesian VAR models which we employ. The results from our empirical analysis are presented and discussed in Section 3. Finally, Section 4 concludes.

2. The Bayesian VAR models

We rely on BVARs for our analysis since the inflation equation of the BVAR can be seen as a “dynamic generalization of the Phillips curve” (King and Watson, 1994, p. 172). While we pay special attention to the inflation equation, our analysis is mainly based on the full bivariate system since important aspects of the dynamic relation between the variables otherwise could be lost.

Defining the vector of dependent variables as \( \mathbf{z} \) = \( \begin{bmatrix} \Delta p \end{bmatrix} \), where \( \Delta p \) is the unemployment rate and \( p \) is inflation – we specify the BVAR with time-varying parameters and stochastic volatility (TVP-SV) in “structural” form as

\[
\Delta p = \begin{bmatrix} \Theta_{1} & \Theta_{2} \\ \Theta_{3} & \Theta_{4} \end{bmatrix} \begin{bmatrix} \Delta p_{t-1} \\ \Delta p_{t-2} \end{bmatrix} + \cdots + \begin{bmatrix} \Theta_{n+1,1} & \Theta_{n+1,2} \\ \Theta_{n+1,3} & \Theta_{n+1,4} \end{bmatrix} \begin{bmatrix} \epsilon_{t-1} \\ \epsilon_{t-2} \end{bmatrix} + \epsilon_{t}
\]

(1)

where \( \Theta_{n} \) is a 2x2 lower triangular matrix with ones on the diagonal, \( \Theta_{n} \) contains the time-varying intercepts and the matrices \( \Theta_{n} \) describe the dynamics. The vector of disturbances, \( \epsilon_{t} \) follows \( \epsilon_{t} \sim N(0, \Sigma) \) where \( \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \). Lag length is set to 4. Collecting the free parameters of \( \Theta_{n} \) and \( \Sigma \) in the 19x1 parameter vector \( \Theta \) we specify the processes for the time-varying parameters and log-volatilities as random walks:

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3 It can be noted that it is in the sense that \( \Theta_{n} \) is a diagonal matrix that the model is “structural”. This interpretation of “structural” – used by, for example, Chan and Eisenstat (2018) – is different from the literature using “structural VARs” in order to identify “structural shocks” such as supply shocks or monetary policy shocks; see Bernanke (1986) or Blanchard and Quah (1992) for early contributions. We have no such ambitions in this paper.
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\mathbb{E}_{-1} = \mathbb{E}_{-1} + \mathbb{E}_{-2}
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\[
\mathbb{E}_{-1} = \mathbb{E}_{-1} + \mathbb{E}_{-2}
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(2) (3)

where \( \mathbb{E}_{-1} \), \( \mathbb{E}_{-1} \), and \( \mathbb{E}_{-2} \). When estimating the BVAR with constant parameters and covariance matrix, the variances \( \mathbb{E}_{-1} \) and \( \mathbb{E}_{-2} \) are restricted to be zero.

When it comes to the issue of model selection – that is, assessing whether the TVP-SV model or the model with constant parameters and covariance matrix is preferred by the data – we rely on marginal likelihoods. In a Bayesian setting, this is the appropriate measure of how well the model and prior agrees with the data; the model with the highest marginal likelihood is the one preferred by the data. We tailor the prior to match the scale and variation of the data as this is crucial for the model comparison with marginal likelihoods. For the constant parameter BVAR we use a diffuse prior for the regression parameters, \( \mathbb{E}_{-1} \), \( \mathbb{E}_{-1} \), and inverse Gamma priors for the diagonal elements of \( \mathbb{E}_{-1} \) with \( \mathbb{E}_{-1} = 5 \) and \( \mathbb{E}_{-1} \) selected to match the prior mean with the residual variance from univariate AR-models.

For the TVP-SV BVAR the same diffuse normal prior is used for the initial condition, \( \mathbb{E}_{-1} \). The diagonal elements of \( \mathbb{E}_{-1} \) have inverse Gamma priors, \( \alpha \mathbb{E}_{-1} \) \( \mathbb{E}_{-1} \mathbb{E}_{-1} \) with \( \mathbb{E}_{-1} = 5 \) and prior means of 0.01 for the intercepts and 0.0001 for the other regression parameters. For the time-varying variances, the prior for the initial condition is selected to closely resemble the prior for the constant variance case. The initial condition for the log-variances has a normal prior, \( \mathbb{E}_{-1} = 0.25 \) \( \mathbb{E}_{-1} \), that is the variance is log-normal and the elements of \( \mathbb{E}_{-1} \) are selected so that the prior means of \( \mathbb{E}_{-1} \) coincides with the constant variance case. Finally, the prior for the diagonal elements of \( \mathbb{E}_{-1} \) is inverse Gamma with shape parameter 5 and mean 0.01.

For posterior inference, we rely on the Markov Chain Monte Carlo-sampler employed by Chan and Eisenstat (2018).

3. Empirical findings

3.1 Price inflation

We use data on seasonally adjusted unemployment rate and CPIF\(^4\) inflation ranging from 1995Q1 to 2017Q3. CPIF inflation is calculated as \( \frac{\text{CPIF}_{t-1}}{\text{CPIF}_{t-2}} = 100 \), where \( \text{CPIF}_{t} \) is the CPIF index at time \( t \).

\(^4\) CPIF is the consumer price index with a fixed interest rate.
This is a reasonable starting point since it formally constitutes the start of Sweden’s inflation targeting regime.\(^5\) Data are shown in Figure 1.

Estimating the two models, we find that the log marginal likelihood is -124.0 for the model with constant parameters and covariance matrix and -117.9 for the model with time-varying parameters and stochastic volatility.\(^6\) The Bayes factor in favour of time-varying parameters and stochastic volatility is 464 and the evidence in favour of the latter model is “decisive” using the terminology of Kass and Raftery (1995). We accordingly conclude that the Phillips curve has not been stable.

**Figure 1. Unemployment rate and price inflation.**

![Unemployment and Price Inflation Chart](chart.png)

Note: Variables measured in percent. Price inflation is given as the year-on-year change in the CPIF index.
Source: Macrobond

We next turn our attention to the properties of the Phillips curve given the established presence of time variation; that is, we look at the TVP-SV model. An issue of key interest in a Phillips curve framework is the effect that shocks to the unemployment rate has on inflation. This impulse-response function is given in Figure 2;\(^7\) the other impulse-response functions are shown in Figures A1 to A3 in the Appendix. An unexpectedly high unemployment rate tends – in line with our expectations – to decrease inflation at short horizons (apart from the three-quarter horizon). It can be noted though that the impulse-response function is reasonably stable over time despite the fact that the estimated parameters of the model show time-variation.

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\(^5\) The regime was declared in 1993 but was to start taking place from 1995; see Sveriges Riksbank (1993).

\(^6\) For details regarding the marginal likelihood calculations, see Chan and Eisentstat (2018).

\(^7\) The shock size is one standard deviation. This varies between 0.10 and 0.16 percentage points; see Figure A1 in the Appendix.
Figure 2. Impulse-response function: Effect of shocks to the unemployment rate on price inflation.

Note: Based on bivariate TVP-SV BVAR with $\theta = \theta$. Size of impulse is one standard deviation. Effect in percentage points on vertical axis. Horizon in quarters and time on horizontal axes.

While this impulse-response function looks stable over time, this does not imply that the parameters of the model have been constant. In Figure 3 we present the parameters of the inflation equation when the model has been expressed in the more commonly used “reduced form”

$$\Phi_0 + \Phi_1 Z_{t-1} + \cdots + \Phi_p Z_{t-p} + \epsilon_t$$

where $\Phi_0$, $\Phi_1$, $\Phi_2$, and $\epsilon_t$. As can be seen, there has been a fair bit of variation in the parameters.

The sum of the coefficients of lagged unemployment in Figure 3 provides a measure of the “slope” of the Phillips curve. This is plotted in Figure 4. Judging by the 68 percent credible interval, the slope of the Phillips curve has not changed dramatically during the sample. Looking at the point estimate, the story is somewhat different as it ranges from -0.11 to -0.37. Interestingly, the slope has not been unusually small between 2011 and 2016 indicating that Sweden’s low inflation in this period cannot be explained by a flat Phillips curve.

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8 This is a common definition of the slope of the Phillips curve; see, for example, Knotek and Zaman (2017) and Karlsson and Österholm (2018).